Gradient-based Worst Case Search Algorithm for Robust Optimization

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The impact of manufacturing and assembly tolerances is a very critical aspect in the analysis of electromagnetic devices performance [1]. When available, the worst solution (WS) $\mathbf{x_W}$ in the tolerance domain $\mathrm{D}_{\mathbf{Tx_0}}$ around a design solution $\mathbf{x_0}$ is an effective index of possible performance degradation due to tolerances [2]-[4]. Such information is very useful also at each step of an optimization process, if the robustness should be included in the iterative optimization procedure. In this case the promptness of WS evaluation is very critical.
This paper proposes to take advantage of the possible knowledge of the gradient of both the performance and tolerance constraint functions. Here, an effective technique is proposed to provide a reliable estimation of the gradient when the derivatives of the objective function are not available.

Let's assume that $\mathbf{x_0}$ is a solution of an optimization problem

$$\mathbf{x_0}: \quad \min_{\mathbf{x} \in \mathrm{D_R}} f_{obj}(\mathbf{x}) \tag{1}$$

where $\mathrm{D_R}$ is the research domain in R^N. The worst impact of the tolerances on the device performance can be looked for by facing with a maximization problem on the function $f_{obj}(\mathbf{x})$ near the design point $\mathbf{x_0}$:

$$\mathbf{x_W}(\mathbf{x_0}): \quad \max_{\mathbf{x} \in \mathrm{D}_{\mathbf{Tx_0}}} f_{obj}(\mathbf{x}) \tag{2}$$

where the condition $\mathbf{x} \in \mathrm{D}_{\mathbf{Tx_0}}$ can possibly be expressed by

$$\mathrm{c}_{\mathbf{x_0}}(\mathbf{x}) \leq 0. \tag{2'}$$

Of course, assuming $\mathrm{D}_{\mathbf{Tx_0}}$ fully included in $\mathrm{D_R}(\mathrm{D}_{\mathbf{Tx_0}} \cap \mathrm{D_R} = \mathrm{D}_{\mathbf{Tx_0}})$, it follows that $\mathrm{c}_{\mathbf{x_0}}(\mathbf{x_W}) = 0$ because $\mathbf{x_W} \in \partial \mathrm{D}_{\mathbf{Tx_0}}$. If both $f_{obj}(\mathbf{x})$ and $c(\mathbf{x})$ are smooth enough, the "worst case" condition is characterized by the alignment of the gradients' direction of both f_{obj} and c (see fig. 1 for an example):

$$\nabla f_{obj}(\mathbf{x_W}) = k \nabla \mathrm{c}_{\mathbf{x_0}}(\mathbf{x_W}), \quad k \neq 0. \tag{3}$$

The mathematical formulation (1)-(3) can be applied in two different cases:

a) in order to evaluate the robustness of a design solution, from a minimization process of the objective function;

b) within an optimization process, in order to look for a solution characterized by a suitable robustness.

Within the limits of a suitable smoothness, f_{obj} can be approximated by a second order Taylor expansion inside $\mathrm{D}_{\mathbf{Tx_0}}$:

$$f_{obj}(\mathbf{x}) \cong \varphi(\mathbf{x}) = \frac{1}{2}\mathbf{x}^{\mathbf{T}}\mathbf{H}\mathbf{x} + \mathbf{g}^{\mathbf{T}}\mathbf{x} + f_0. \tag{4}$$

In this case, the required property of convexity in $\mathrm{D}_{\mathbf{Tx_0}}$ is automatically satisfied by (4). In addition, the gradient of $\varphi(\mathbf{x})$ can be simply derived as follows:

$$\nabla f_{obj}(\mathbf{x}) \approx \mathbf{H}\mathbf{x} + \mathbf{g}. \tag{5}$$

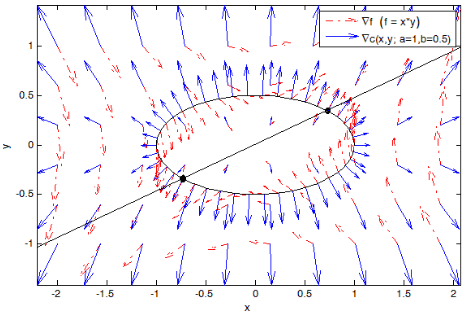

Fig. 1. Example of tolerance gradients alignment

As a matter of fact, since **H** is the Hessian matrix of f_{obj}, in the limits of Schwarz theorem validity, it is characterized by just N(N+1)/2 independent elements which can be evaluated by using a best-fitting technique. As an important byproduct, in the robust optimization applications, the direct knowledge of the quadratic approximation of $\varphi(\mathbf{x})$ guarantees a very effective use of the Newton minimization algorithms, as it provides in a direct way the next step of the iteration process.
In addition, in order to obtain a linear expression for (3), also the function $\mathrm{c}_{\mathrm{x}_0}(\mathbf{x})$, which in (2') describes the tolerance domain, should fall in the class of quadratic function. The problem can be effectively faced with by replacing the brick with a suitable N-dimensional hyper-ellipse, centered in $\mathbf{x_W}$:

$$c(\mathbf{x}) = \frac{\left(\mathrm{x}_1 - \mathrm{x}_{\mathrm{W}_1}\right)^2}{(\mathrm{a}_1)^2} + \frac{\left(\mathrm{x}_2 - \mathrm{x}_{\mathrm{W}_2}\right)^2}{(\mathrm{a}_2)^2} + \cdots + \frac{\left(\mathrm{x}_\mathrm{N} - \mathrm{x}_{\mathrm{W}_\mathrm{N}}\right)^2}{(\mathrm{a}_\mathrm{N})^2} - 1 \tag{6}$$

able to fit in a suitable way the tolerance ranges. The WS is the solution of (3) with the constraint:

$$\mathrm{c}_{\mathrm{x}_0}(\mathbf{x_W}) = 0. \tag{7}$$

If an "elliptically shaped" boundary is assumed, then (7) is a quadratic equation that must be satisfied together with (3), making the (3)-(7) system non-linear. However, a very effective way to preserve the benefits of linearity is to look for solutions of eqs. system (3) only, but as a function of the *k* parameter. In this way an infinite set of solutions is found. In practice $\mathbf{x_W}$ can be obtained by solving the linear system (3) by assuming a generic value for the parameter *k*, e.g. *k*=1. Finally, two worst candidates are provided and the ranking of those solutions simply provides the actual WS.

References

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