HOMOGENIZATION TECHNIQUE FOR INTERCONNECT MODELING BASED ON THE MONOTONICITY PROPERTY

A. Maffucci[1], A.Tamburrino [1,2], A. Vento[1], S. Ventre[1]

[1]DIEI, Università di Cassino e del Lazio Meridionale, Via Di Biasio 43, 03043, Cassino (FR)
[2]ECE, Michigan State University, East Lansing, USA

The homogenization techniques are widely used in the numerical electromagnetism to reduce the computational cost of simulations and to get a thorough insight into the qualitative behavior of materials. The purpose of the work is to model a 3D electrical interconnect embedded in an inhomogeneous dielectric in terms of an equivalent, simplified, 2D transmission line embedded in a homogeneous dielectric. The procedure estimates the values of the equivalent per-unit-length parameters via homogenization of physical and geometrical parameters, such as permittivity and inter-conductor distance. The homogenization exploits the property of Monotonicity, initially proposed for the imaging of materials.

The starting point is represented by the knowledge of any input-output representation of the inhomogeneous interconnect (e.g Z matrix or S parameters), either given by measurements or simulations. Then, an inverse problem based on the theory of TL is applied, to relate such data to equivalent RLC parameters [1]. This association is valid is the operation frequency is low enough to consider the interconnect to be electrically small. If this is true, the RLC parameters are taken as a target for an identification procedure, where the parameters of the equivalent homogeneous transmission line (longitudinal length, inter-conductor distance and permittivity) are varied to find the optimal solution.

The first parameter to be identified is the equivalent longitudinal length of the line d_{eq}. Being $\underline{\underline{\tilde{R}}}$ the target resistance matrix obtained by solving the inverse problem before, and $\underline{\underline{R}}'$ the p.u.l. resistance matrix associated to the simplified TL model (diagonal matrix, whose generic element is $\underline{\underline{R}}' = diag(\rho / A)$, where ρ and A are respectively the resistivity and the cross section area of the conductor), we assume that d_{eq} is the value that minimizes the square Frobenius norm of $\underline{\underline{\tilde{R}}} - \underline{\underline{R}}' d_{eq}$. The minimum is obtained by:

$$d_{eq} = \frac{tr\left(\underline{\underline{\tilde{R}}}\,\underline{\underline{R}}'\right)}{tr\left(\underline{\underline{R}}'^2\right)} \qquad (1)$$

Once this parameter is known, it is possible to identify the equivalent inter-conductor distance h_{eq} and the equivalent permittivity ε_{eq} by exploiting the property of Monotonicity [2]. This procedure follows some steps. First of all, we have to choose the quantity to which that property will be applied. In that case, the electrostatic capacitance matrix has been chosen, because it is monotonical w.r.t. the two parameters to identify. A set of matrices $\underline{\underline{C}}' = \underline{\underline{C}}'(\varepsilon, h)$ is computed for different values of the pair *(ε,h)*. From the knowledge of the physical structure of the interconnect, we assume to know the bounds for these two parameters, hence we seek for the solution falling into a domain *W* so that: $(\varepsilon, h) \in W = [\varepsilon_{\min}, \varepsilon_{\max}] \times [h_{\min}, h_{\max}]$. Then, being $\underline{\underline{\tilde{C}}}$ the target capacitance matrix obtained by the inverse problem described above, the eigenvalues of the matrix $\underline{\underline{\Delta C}}(\varepsilon,h)=\underline{\underline{\tilde{C}}}-\underline{\underline{C}}'d$ are computed and the sign index is calculated as $S_i(\varepsilon,h)=\dfrac{\sum_i \lambda_i(\varepsilon,h)}{\sum_i |\lambda_i(\varepsilon,h)|}$. The set of admissible solutions, complying with the Monotonicity Property, is $S_i(\varepsilon,h)<1$, while $S_i(\varepsilon,h)=1$ is the forbidden region. It can be demonstrated that the solutions taken on the boundary between the two regions given the minimum of the energy associated to the difference matrix $\Delta\underline{\underline{C}}$. A case study is illustrated:

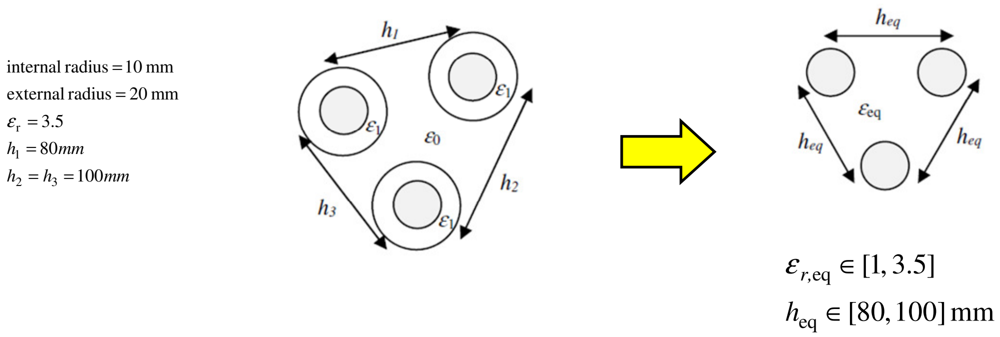

Figure 1. The considered case study.

The starting interconnect is that depicted on the left: a bundle of three conductors in an inhomogeneous dielectric. The equivalent simplified transmission line is on the right.

The capacitance matrix for the configuration on the left has been extracted by means of COMSOL. The p.u.l. capacitance matrix for the simplified system has been obtained by using semi-analytical formulas [3]. In the next figure, the sign index has been plotted.

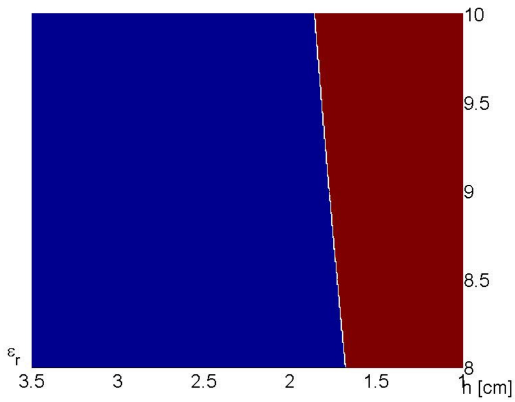

Figure 2. Map of the sign index.

The blue region corresponds to the admissible region, while the red one is the forbidden region. The solutions taken on the boundary correspond to the minimum of the energy of the difference matrix.

References

[1] G.Miano, A.Maffucci, *Transmission Lines and Lumped Circuits*, Academic Press, 2001

[2] A.Tamburrino and G.Rubinacci, "Fast Methods for Quantitative Eddy Current Tomography of Conductive Materials", IEEE Trans. on Magnetics, vol. 42, pp.2017-2028, Aug. 2006

[3] C.R. Paul, *Analysis of Multiconductor Transmission Lines*, Wiley, 2008