# Three-dimensional Computation of Magnetic Field in Hysteretic Media with Time-Periodic Sources

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# Introduction

This paper presents a 3D integral formulation for the efficient numerical modelling of the 3D magnetic field in hysteretic media with time periodic sources. The numerical model is suitable for simulating the signal from the (non-destructive) micromagnetic characterization (of mechanical properties) in metallic materials. In particular, the problem of estimating the incremental permeability is considered.

Material characterization is a field of paramount importance in the steel industry. Obtaining information on the mechanical properties of a specimen without resorting the expensive destructive tests is a major challenge for electromagnetic non-destructive evaluation methods. Usually, the micromagnetic characterization of mechanical properties is achieved using the measurements of the incremental permeability along a hysteresis loop, multifrequency eddy currents induced in the hysteretic medium, Barkhausen noise, and harmonic analysis [1]. The design and optimization of the probes, as well as a reliable interpretation of the measurements, require accurate numerical models. Among this, a key role is played by the magneto-quasi-stationary Maxwell equations in hysteretic media with time periodic sources. Here, the integral method presented in [2] is extended to include an efficient numerical modelling of incremental permeability and harmonic analysis. The information related to the incremental permeability curve, together with other electromagnetic measurements like the harmonic content of the field, can be fed into a multiple regression approach as a tool to model material characteristics [4].

#### Numerical Model and results

The integral model arises from imposing in weak form the electric and magnetic constitutive relationships, under the magneto-quasi static approximation of the Maxwell equations. The formulation considers a vector hysteresis model of the constitutive relationship of the magnetic material. Here, it is adopted an isotropic vector generalization of the classical Jiles-Atherson model [3], which has been considered in the analysis of NDE configurations in [2]. The solution of this time-periodic nonlinear set of equations with hysteretic media cannot be obtained within a reasonable CPU time in the time domain. For this reason, it is used a frequency-domain iterative algorithm converging to the steady-state solution of the problem [5] [6] [7]. The steady-state solution is computed by approximating the unknowns fields by using a truncated Fourier series. The whole problem is solved using the iterative fixed point procedure described in [2]. The numerical implementation has been carried out in a parallel environment, with a fast computation (sparsification) of the relevant matrix-by-vector products involving full matrices [8], [9].

The effectiveness of the numerical model has been proved in [2], here it is considered the micromagnetic characterization of mechanical properties using the measurements of a quantity related to the incremental permeability along a hysteresis loop and harmonic analysis. The system (Figure 1) is composed of a soft magnetic yoke ( $\mu_r$ =2000) with two low-resistivity excitation coils, driven by a low frequency (f = 200 Hz) voltage V, and a ferromagnetic plate (500mm × 500mm × 1 mm). In addition, there is a small excitation circular coil (mean radius r = 3 mm) powered at high

frequency (f > 20 kHz) and placed between the legs of the yoke with a liftoff of 0.5mm and a



Figure 1: Left: sketch of the NDE testing system. Right: its geometry and discretization: the low frequency coils.



Figure 2: Plot of the magnetic field at t = 0 s



Figure 3: the tangential magnetic field just under the high frequency coil as a function of time (solid line), its first harmonic (dashed line), and its upper harmonics (dashed dot line)



Figure 4: The evolution with the tangential magnetic field of the diagonal incremental permeability matrix in an element of the plate under the detection coil (V = 20 V)



Figure 5: the signal evolution in the detection coil (V = 20 V)

detection coil in its proximity. A Hall sensor placed under the small coil in the close proximity of the ferromagnetic plate measures the tangential magnetic field. Since the field produced by the HF coil is much smaller than the field produced by the LF excitation, it can be considered as a perturbation around the operating point imposed by the LF coil. Then, the HF perturbation can be computed by solving a proper linearized problem for each operating point on the LF periodic response. Figure 2 shows a snapshot of the magnetic field right beneath the high frequency coil as a function of time and its harmonic content is shown. This harmonic content can be used for estimating several useful quantities for NDT [1] such as the material yield strength. Figure 4 shows the incremental permeability along different axes. Eventually, figure 5 shows the signal at the detection coil, thus proving that it can be used to infer the values of the incremental permeability in the material, as it is usually assumed by experimentalists.

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