Numerical Behaviour of Models of Composite Materials in E'NDT at Low Frequencies

L.Barbato[1], S.Minucci[2], G. Rubinacci[2], A. Tamburrino[1,3], A. Vento[1], S. Ventre[1]

[1]DIEI, Università di Cassino e del Lazio Meridionale, Via G. di Biasio 43, Cassino (FR), Italia
[2]DIETI, Università degli Studi di Napoli Federico II, Via Claudio 21, Napoli, Italia
[3]ECE, Michigan State University, East Lansing, Mi 48824, USA, e-mail: tamburrino@unicas.it

Keywords: Composite Materials, Low-Frequency Breakdown, Nondestructive Testing

Introduction

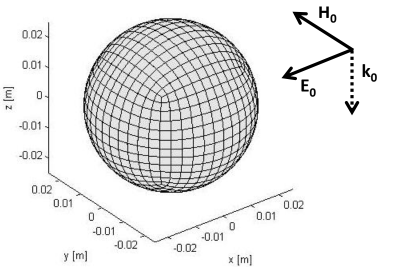
In this paper the numerical modelling of composite materials in view of non-destructive testing (NDT) is considered. The numerical modelling is critical especially in the "low-frequency" regime because of a strong ill-conditioning of the relevant stiffness matrix. The proposed numerical model is robust with respect to this underlying ill-conditioning.

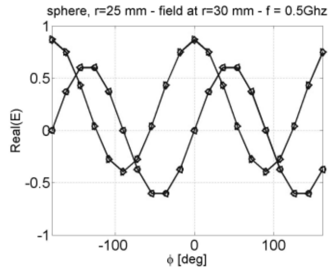
The conductivity of new composite materials, specifically Carbon Fiber Reinforces Polymers (CFRPs), is reasonably high for allowing nondestructive evaluation based on eddy currents testing. However, the complex electrical structure of the CFRP composites, characterized by higher electrical conductivity along the direction of the carbon fibers and lower conductivity perpendicular to the fibers, poses very difficult problems for their electromagnetic description and modelling. The classic electromagnetic models based on the magnetic (electric) field integral equations, may fail to provide the correct solution at low frequencies such as those of NDT applications [1], [2], [3]. This is due to the low-frequency break-down problem consisting of a strong ill-conditioning of the relevant stiffness matrix. This strong ill-conditioning is due to the different scaling of the solenoidal and non-solenoidal components of the unknown fields, w.r.t. the frequency.

Numerical Model and results

The proposed numerical model has its foundation in: (i) the magneto-quasi static model [4] based on a volume integral formulation in term of the solenoidal current density, (ii) the full wave model presented in [5] and (iii) a numerical model for plasmonic analysis [6]. Specifically, the proposed model merges the approaches of [5] and [6] to extend the analysis to composite materials described by a homogenized constitutive relationship. The constitutive relationship can be assumed to be linear but anisotropic, i.e. it is described by a proper permittivity tensor $\boldsymbol{\varepsilon}$ (no magnetic properties are considered). The unknown of the numerical model is the polarization current density. Moreover, since electric charges may be present as surface and volume charge density, the unknown current density is split into three components (loop-star-facet decomposition), a purely solenoidal one and the other two taking into account surface and volume charge densities. These separate components (the loop, star and facet components) can be properly scaled for achieving a better conditioned numerical model from almost DC to high frequencies.

For the numerical examples, the scattering from two objects is analyzed. The first object is a sphere of radius 5cm (Figure 1). This scatterer is considered for validating the numerical model since an analytical (Mie [7]) solution exists. For this validation it is assumed an isotropic material with $\varepsilon=8\varepsilon_0$. The applied field is plane wave at 0.5GHz. Figure 2 and 3 shows the comparison between the numerical and the analytical results (non-vanishing components of the scattered field). The second object (Figure 4) refers to a composite dielectric material having dimension (10 cm $\times$ 10cm $\times$ 2.5cm) and a diagonal dielectric permittivity tensor given by $\varepsilon_0$diag(4+0.5j,3,3). In this second case, it is also considered the presence of a volumetric anomaly. The applied field is a plane wave 1GHz. Figure 5-6 and Figure 7-8 show the relative results (non-vanishing components of the scattered field). The same figures include also the case of an isotropic medium $\varepsilon_0(4+0.5j)\mathbf{I}$ ($\mathbf{I}$ is identity matrix).





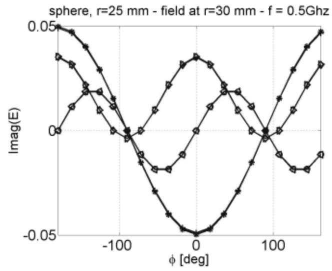


Figure 1: Test sphere mesh and incoming plane wave

Figure 2: Real part of the scattered field

Figure 3 Imaginary part of the scattered field

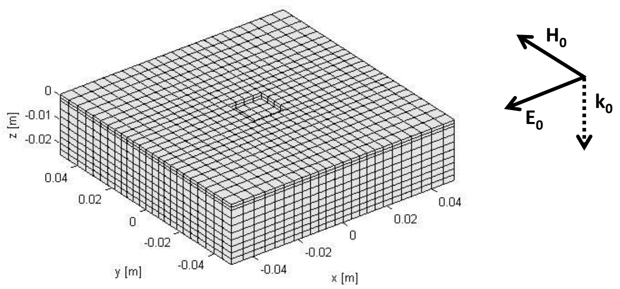


Figure 4: Finite element mesh of the parallelepiped under microwave NDT operations together with the description of the incoming wave. A volumetric surface breakinh defect is considered.

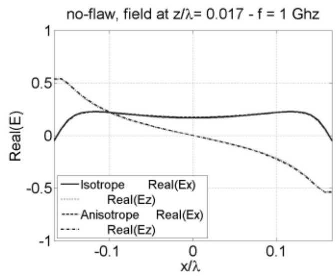
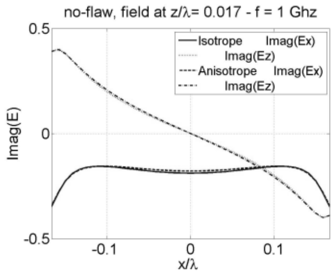
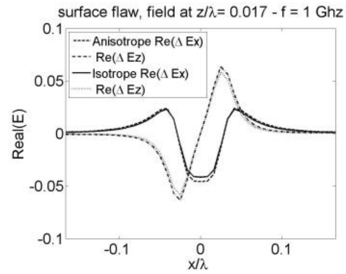


Figure 5: Real part of the scattered field (defect free configuration).





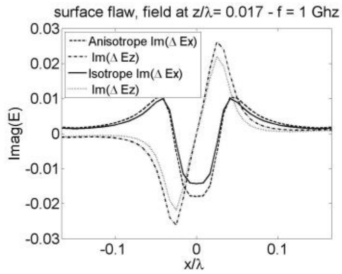


Figure 6: Imaginary part of the scattered field (defect free configuration).

Figure 7: Real part of the anomalous scattered field due to the volumetric defect

Figure 8: Imaginary part of the anomalous scattered field due to the volumetric defect

Conclusions

A full-wave numerical formulation suitable for broadband computation has been presented. Specifically, the integral formulation exploits a proper splitting of the unknown polarization current density in order to obtain for each component a different behaviour w.r.t the frequency. In this way, they can be easily scaled to mitigate the ill-conditioning of the stiffness matrix. The model has been successfully validated and applied to a NDT-like test case. This work was supported in part from the European Community’s Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 285549 and by the Association EURATOM/ENEA/CREATE.

References

[1] J.S. Zhao, and W. C. Chew, *IEEE Trans. Antennas Propag.* **48** (2000), 1635–1645
[2] Y. Zhang, T.J. Cui, W.C. Chew, J.S. Zhao, *IEEE Trans. Antennas Propag.* **51** (2003), 1864–1871
[3] I.T. Chiang, W.C. Chew, *J. of Electromagn. Waves and Appl.* **19** (2005), 289-306.
[4] L.Barbato, C. Petrarca, G. Rubinacci, A. Tamburrino and S. Ventre, 17th International Workshop on Electromagnetic Nondestructive Evaluation, Rio de Janeiro (Brazil), July 29 – August 1, 2012, pp. 52.
[5] G.Rubinacci and A. Tamburrino, *IEEE Trans. Antennas Propag.*, **54** (2006), 2977- 2989
[6] G.Miano, G. Rubinacci, A. Tamburrino, F. Villone, *IEEE Trans. Magn.*, **44** (2008):, 822-825.
[7] J.A. Stratton, *Electromagnetic Theory*, McGraw Hill, New York, NY, 1941, Vol. 32, n° 8, pp. 798–818.