Vertical Displacement Events: Dynamics and consequences

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Introduction

• VDEs:
  – Vertical Displacement Events may occur for elongated, vertically unstable plasmas
  – Elongated plasmas are stabilized by active control systems
  – However, in some cases, the installed power is not sufficient to keep the plasma stable
  – An instability (triggered by some perturbation, e.g. an ELM) grows up exponentially
  – The plasma motion is typically axisymmetric and vertical
  – Eventually the plasma hits the wall, with consequent thermal loads on the wall, rise of halo currents, and finally plasma current quench
  – These disruptions are quite dangerous because the particular nature of the mechanical loads.
  – Even more dangerous are the Asymmetric VDEs because of the concentration of the loads.

This lecture is focused on:

• plasma modelling for the analysis and the control of the vertical instabilities:
  – full MHD equations
  – rigid displacement
  – perturbed equilibrium

• phenomenological description of VDEs and their effects
Elongated plasmas (1)

- To guarantee radial equilibrium an external vertical field has to be applied according to virial theorem: a sector is subjected to a net radial force due to the total pressure $p + B^2/2\mu_0$ from the rest of the plasma.

\[
B_V = \frac{\mu_0 I_p}{4\pi R_0} \left[ \ln \frac{8R_0}{a} - \frac{3}{2} + \beta_{pol} + \frac{\ell_i}{2} \right],
\]

\[
I_p = \int_{\Omega_r} J_\psi d\Omega, \quad \beta_{pol} = 4\int_{V_r} p dV / \mu_0 R_0 I_p^2, \quad \ell_i = 4\int_{V_r} \left( \frac{B^2}{2\mu_0} \right) dV / \mu_0 R_0 I_p^2
\]

- A large $R_0/a$ aspect ratio plasma would naturally tend to have a circular cross section: $b/a \approx 1$ with a uniform external vertical field (slightly larger for common values of $R_0/a$, e.g. $b/a = 1.05$).

\[
F_{ext} = \int_{V_p} J \times B_{ext} dV
\]

\[
dF_{ext} \approx B_V I_p R_0 d\varphi
\]

orientation according to the standard right-hand-rule.
Elongated plasmas (2)

- Vertically elongated plasmas ($b/a > 1$), have to be used to maximize the performance-to-cost ratio (large volume and pressure).

- To elongate the plasma an additional force distribution is needed (with zero total force):

$$\Delta F_U + \Delta F_L = 0$$

- This is obtained with a quadrupole field:

$$\Delta F_U + \Delta F_L = 0$$

![Diagram showing vertical field and quadrupole field](image-url)
Elongated plasmas (3)

- **Field index:**
  \[ n = -\frac{R_0}{B_V} \frac{\partial B_r}{\partial z} \]

  - a slight plasma displacement directed upwards \( \delta z > 0 \) would give rise to a net force directed upwards \( \delta F_z > 0 \) → instability

  \[ \delta F_z \approx -2\pi R_0 \delta B_r I_p \]

  \[ \delta F_z \approx 2\pi n B_V I_p \delta z \]

  - active stabilization needed
MHD instabilities (1)

- ideal MHD assumptions
  - single plasma fluid
  - $\sigma \to \infty$ and $E_i \to 0$ in the plasma
  $\Rightarrow E + v \times B = 0$

MHD Model

quasi-stationary Maxwell’s equations

\[
\nabla \times E = -\frac{\partial B}{\partial t} \\
\nabla \times H = J \\
\nabla \cdot B = 0
\]

with constitutive laws:

\[
B = \mu H \\
J = \sigma \left( E + v \times B + E_i \right)
\]

thermo-fluid-dynamics

\[
\frac{D \rho}{Dt} + \rho \nabla \cdot v = 0 \\
\rho \frac{D v}{Dt} = J \times B - \nabla p \\
\frac{D}{Dt} \left( \rho p^{-\gamma} \right) = 0
\]
MHD instabilities (2)

- MHD equilibrium assumptions:
  - stationary conditions: \( \partial / \partial t = 0 \)
  - static conditions: \( \mathbf{v} = 0 \)

\[ \mathbf{J} \times \mathbf{B} = \nabla p \]

equilibrium equation

\( p \): kinetic pressure

\[
\begin{align*}
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{H} &= \mathbf{J} \\
\n\nabla \cdot \mathbf{B} &= 0
\end{align*}
\]

with constitutive laws:

\[ \mathbf{B} = \mu \mathbf{H} \]

\[ \mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B} + \mathbf{E}_i) \]

\[
\begin{align*}
\frac{D \rho}{Dt} + \rho \nabla \cdot \mathbf{v} &= 0 \\
\rho \frac{D \mathbf{v}}{Dt} &= \mathbf{J} \times \mathbf{B} - \nabla p \\
\frac{D}{Dt} (\rho p^{-\gamma}) &= 0
\end{align*}
\]

MHD Model

quasi-stationary Maxwell's equations

thermo-fluid-dynamics
MHD instabilities (3)

- Linearized stability equations in the plasma:
  - linearization around equilibrium conditions ($v=0$)
  - basic variable: Eulerian displacement $\delta \xi$

\[
\delta \xi = \int \delta v \, dt \quad \Rightarrow \quad \delta v = \delta \xi
\]

\[
v = 0 \text{ at equil.}
\]

- **FORCE**

  \[
  \nabla \times B = \mu_0 J, \quad \rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = J \times B - \nabla p
  \]

  \[
  \text{Linearized MHD Model}
  \]

  \[
  \nabla \times \delta B = \mu_0 \delta J, \quad \delta \rho \left( \frac{\partial \delta v}{\partial t} + v \cdot \nabla \delta v \right) + \rho \left( \frac{\partial \delta v}{\partial t} + \delta v \cdot \nabla v + v \cdot \nabla \delta v \right) = \delta J \times B + J \times \delta B - \nabla \delta p
  \]

  \[
  \rho \ddot{\delta \xi} = \mu_0^{-1} (\nabla \times \delta B) \times B + \mu_0^{-1} (\nabla \times B) \times \delta B - \nabla p
  \]
MHD instabilities (4)

- **MAGNETIC FIELD**

  \[ MHD \text{ Model: } E + v \times B = 0, \quad \nabla \times E = -\dot{B} \]

  \[ \text{Linearized MHD Model} \]

  \[ \delta E + \delta v \times B + v \times \delta B = 0 \quad \nabla \times \delta E = -\dot{\delta B} \]

  \[ \dot{\delta B} = -\nabla \times \delta E = \nabla \times (\delta \dot{x} \times B) \Rightarrow \delta B = \nabla \times (\delta \dot{x} \times B) \]

  (after time integration with zero initial conditions)

- **PRESSURE**

  \[ \text{MHD Model} \]

  \[ \begin{cases} \frac{D\rho}{Dt} = -\rho \nabla \cdot v \\ D\left( p p^{-\gamma} \right) \frac{Dt}{Dt} = 0 \end{cases} \Rightarrow \dot{p} + v \cdot \nabla p = -\gamma p \nabla \cdot v \]

  \[ \text{Linearized MHD Model: } \delta p + \delta \dot{x} \cdot \nabla p = -\gamma p \nabla \cdot \delta \dot{x} \]  
  (discarding terms multiplied by unperturbed velocity)

  \[ \downarrow \]

  \[ \delta p = -\delta \dot{x} \cdot \nabla p - \gamma p \nabla \cdot \delta \dot{x} \]  
  (after time integration with zero initial conditions)
MHD instabilities (5)

- LINEARIZED FORCE BALANCE IN TERMS OF PLASMA DISPLACEMENT

\[ \rho \ddot{\xi} = \mathbf{F}(\xi) \]

with

\[ \mathbf{F}(\xi) = \mu_0^{-1} \nabla \times [\nabla \times (\xi \times \mathbf{B})] \times \mathbf{B} + \mu_0^{-1} (\nabla \times \mathbf{B}) \times \nabla \times (\xi \times \mathbf{B}) + \nabla (\mathbf{\xi} \cdot \nabla p + \gamma p \nabla \cdot \xi) \]

(F: force operator depending on equilibrium distributions of B and p)

to solve the above PDE equation:
- coupling to Maxwell and circuit equations outside the plasma
- interface conditions at plasma boundary
  - e.g., continuity of magnetic field and pressure at plasma boundary
  - plasma boundary: outermost closed magnetic surface that does not intersect solid walls

simplest case: plasma in contact with a perfect conducting wall:
- \( \mathbf{B} \cdot \mathbf{n} = 0, \mathbf{E} \times \mathbf{n} = 0 \Rightarrow \mathbf{v} \cdot \mathbf{n} = 0 \) hence \( \xi \cdot \mathbf{n} = 0 \)
- PDE system:

\[
\begin{cases}
\rho \ddot{\xi} = \mathbf{F}(\xi) & \text{in } \Omega_p \\
\mathbf{\xi} \cdot \mathbf{n} = 0 & \text{on } \partial \Omega_p
\end{cases}
\]
Stability analysis with the linearized MHD model (1)

• Eigenvalue analysis in the frequency domain using weighted residuals:

PDE problem:
\[
\begin{cases}
\rho \ddot{\xi} = F(\dot{\xi}) \quad \text{in } \Omega_p \\
\dot{\xi} \cdot n = 0 \quad \text{on } \partial \Omega_p
\end{cases}
\]

weighted residuals:
\[
\int_{\Omega_p} w \cdot \rho \dddot{\xi} \, d\Omega = \int_{\Omega_p} w \cdot F(\dot{\xi}) \, d\Omega, \quad \text{with } \dot{\xi} \cdot n = 0 \text{ on } \partial \Omega_p, \quad \forall w
\]

Galerkin's method:
\[
\dot{\xi}(r) = \sum_{k=1}^{N} c_k u_k(r), \quad \text{with } \dot{\xi} \cdot n = 0 \text{ on } \partial \Omega_p, \quad w_i = u_i
\]

ODE problem:
\[
M \ddot{c} = Fc \quad \text{with } \quad M_{ik} = \int_{\Omega_p} u_i \cdot \rho u_k \, d\Omega, \quad F_{ik} = \int_{\Omega_p} u_i \cdot F(u_k) \, d\Omega
\]

eigenvalue problem:
\[
\dot{x} = Ax \quad \text{with } \quad x = \begin{bmatrix} c \\ \dot{c} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ M^{-1}F & 0 \end{bmatrix}
\]

check real part of the eigenvalues of \( A \) matrix
Stability analysis with the linearized MHD model (2)

• Extensively used for the analysis of internal (fixed boundary) instabilities

• External (moving boundary) instabilities (including vertical instability) strongly interacting with the conductors outside the plasma

• Detailed description of external circuits and passive conductors therefore needed especially for \( n=0 \) (axisymmetric) external modes \( \Rightarrow \) complex coupled systems of PDE equations and circuit equations proposed, applied and still being developed

• Simplified treatment of plasma more commonly and efficiently used for the analysis of vertical instabilities:
  – rigid displacement model
  – perturbed equilibrium approach
Rigid displacement model (1)

- Plasma treated as a single filamentary current (or a rigid set of filaments carrying constant currents) with a single d.o.f.: the vertical position $z_p$
- System equations for a filamentary plasma:

$$\dot{\Psi} + RI = U \quad \text{(circuit equations)}$$

$$m_p \ddot{z}_p = -2\pi r_p B_r I_p \quad \text{(vertical force balance)}$$

$I$: PF circuit currents
$\Psi$: fluxes linked with the circuits
$U$: applied voltages (zero for passive circuits)
$R$: resistance matrix
$m_p$, $z_p$, $r_p$: plasma mass, vertical and radial position
$I_p$: plasma current
$B_r$: radial field acting on the filament
Rigid displacement model (2)

- **Linearized equations:**
  \[ \delta \dot{\Psi} + R \delta I = \delta U \]  
  (circuit equations)
  \[ m_p \delta \ddot{z}_p = -2\pi r_p \delta B_r I_p \]  
  (vertical force balance)

- in terms of \( \delta I \) and \( \delta z_p \)... 
  \[ \left( \frac{\partial \Psi}{\partial I} \right) \delta I + \left( \frac{\partial \Psi}{\partial z_p} \right) \delta \dot{z}_p + R \delta I = \delta U \]
  \[ m_p \delta \ddot{z}_p = -2\pi r_p I_p \left( \frac{\partial B_r}{\partial I} \right) \delta I - 2\pi r_p I_p \left( \frac{\partial B_r}{\partial z_p} \right) \delta z_p \]

- where ...
  \[ \frac{\partial \Psi}{\partial I} = L, \quad \frac{\partial \Psi}{\partial z_p} = I_p \frac{\partial M}{\partial z_p} \]
  \[ -2\pi r_p I_p \left( \frac{\partial B_r}{\partial I} \right) = I_p \left( \frac{\partial M}{\partial z_p} \right)^T \]
  \[ -2\pi r_p I_p \left( \frac{\partial B_r}{\partial z_p} \right) = 2\pi n B_V I_p \]
Rigid displacement model (3)

- Linearized equations:

\[
(\partial\Psi/\partial I)\dot{\delta I} + (\partial\Psi/\partial z_p)\dot{\delta z}_p + R\delta I = \delta U
\]

\[
m_p \delta \ddot{z}_p = -2\pi r_p I_p (\partial B_r/\partial I) \delta I - 2\pi r_p I_p (\partial B_r/\partial z_p) \delta z_p
\]

rewritten as:

**LINEARIZED RIGID DISPLACEMENT MODEL**

\[
L\delta \dot{I} + f \delta \ddot{z}_p + R\delta I = \delta U
\]

\[
m_p \delta \ddot{z}_p = g^T \delta I + F' \delta z_p
\]

\(\delta I\): variation of circuit currents (note \(\delta I_p = 0\))

\(\delta z_p\): variation of plasma vertical position

\(L, R\): circuit inductance & resistance matrix

\(f=g\): stabilizing efficiencies \(I_p(\partial M/\partial z_p)\)

\(m_p\): plasma mass

\(F'\): destabilizing force \(2\pi n B_v I_p\)
Rigid displacement model (4)

- Massy vs massless plasma model

**MASSY MODEL**

\[-(m_p L / F') \ddot{\delta I} - (m_p R / F') \dot{\delta I} + \left( L - g g^T / F' \right) \delta I + R \delta I = \]
\[= -(m_p / F') \ddot{\delta U} + \delta U\]

**MASSLESS MODEL**

\[\left( L - g g^T / F' \right) \dot{\delta I} + R \delta I = \delta U\]

- IMPLICATIONS

\[P(\lambda) = -(m_p L / F') \lambda^3 - (m_p R / F') \lambda^2 + \left( L - g g^T / F' \right) \lambda + R = 0\]

there exist a real unstable eigenvalue if \( F' > 0 \) and \( R > 0 \):

\((m_p \text{ and } L \text{ intrinsically positive})\)

\[P(0) = R > 0\]

\[P(\lambda) \approx -(m_p L / F') \lambda^3 \rightarrow -\infty \text{ for } \lambda \rightarrow +\infty\]
Rigid displacement model (5)

Sample case: eigenvalues with a single circuit

\[ P(\lambda) = -(m_p L / F') \lambda^3 - (m_p R / F') \lambda^2 + (L - g^2 / F') \lambda + R = 0 \quad \text{(MASSY MODEL)} \]

no-wall limit: \( F' \leq 0 \), i.e., \( n \leq 0 \)

no-wall growth rate (\( g = 0 \)): \( \gamma = \lambda_u \approx (F'/m_p)^{1/2} \)

ideal (\( R = 0 \)) wall limit: \( L \leq g^2 / F' \)

stability margin: \( m_s = g^2 / LF' \)

\[ P(\lambda) = (L - g^2 / F') \lambda + R = 0 \quad \text{(MASSLESS MODEL)} \]

growth time and growth rate on the resistive time scale:

\[ \tau_g = (m_s - 1) L / R, \quad \gamma = 1 / \tau_g \]

massless model applicable only for \( F' \leq 0 \) or \( m_s \geq 1 \)

with \( F' > 0 \) and \( m_s < 1 \)

Alfven instability artificially hidden

\[ F' = 2\pi n B v I_p, \quad m_p = \langle \rho \rangle V_p \]

\( n \approx 1, B_v = 1T, I_p = 3 \text{ MA}, \)

\( \langle \rho \rangle = 10^{-6} \text{ Kg/m}^3, V_p = 100 \text{ m}^3 \)

\( \gamma = 0.5 \cdot 10^{-6} \text{ s}^{-1}, \tau_g = 1 / \gamma = 2 \mu s \)

NO ACTIVE STABILIZATION

FEASIBLE ON ALFVEN TIME SCALE
Rigid displacement model (6)

STABILITY MARGIN IN THE MULTI-CIRCUIT CASE

\[ L^* = L - gg^T / F' \]

*modified inductance matrix*

\[ L^* \delta I + R \delta I = 0 \]

*open loop system: \( \dot{x} = Ax, \)

\[
\begin{cases}
    x = \delta I \\
    A = -(L^*)^{-1} R
\end{cases}
\]

\[ Ax_u = \lambda_u x_u \]

*unstable (growth) mode: \( \gamma = \lambda_u > 0 \)

*two alternative definitions of \( m_s \):

1) \( m_s = \) largest real part of the eigenvalues of \( M= - L^* L^{-1} \)

*(all currents weighted independently from resistance)*

2) \( m_s = - (x_u^T L^* x_u)/(x_u^T L x_u) \)

*(resistances and unstable mode taken into account)*
Rigid displacement model (7)

MAIN LIMITATIONS OF RIGID DISPLACEMENT MODEL

- plasma as a single filament → multifilament rigid plasma
- axisymmetric conductors → coupling to 3D eddy currents
- rigid displacement non-consistent with local MHD equilibrium
  - uncorrect estimation of growth rate (especially for triangular plasmas)
  - cumbersome prediction of the excitation of the unstable mode
  - uncorrect modelling for VS magnetic diagnostics

→ perturbed equilibrium approach
Perturbed equilibrium approach (1)

**Plasma**

Assumptions:
- inertial effects neglected (plasma in equilibrium at each time instant)
- the plasma is axisymmetric, and its equilibrium evolution is determined only by the magnetic field averaged along the toroidal angle;
- plasma current density profile parameterized with 3 degrees of freedom: total plasma current $I_p$, poloidal beta $\beta_{pol}$ and internal inductance $l_i$.

**Circuits and Conducting Structures**

Assumptions:
- The mathematical model for the conducting structures is the standard eddy current model, i.e., the quasi-stationary Maxwell equations ($\partial D/\partial t \rightarrow 0$)
- The time evolution of the coil currents is described by the standard circuit equations (with zero applied voltages for passive circuits)
- The use of integral formulations allows for a unified treatment of circuits and eddy currents (even in the 3D case)
Perturbed equilibrium approach (2)

- usually derived from evolutionary MHD equilibrium:
  - \( d\Psi/dt + RI = U \) (circuit equations)
  - \([\Psi, Y]^T \eta (I, W)\) (Grad-Shafranov constraint in symbolic form)
    - \( I \): including PF circuit currents and the plasma current \( I_p \)
    - \( \Psi' \): fluxes linked with the above circuits
    - \( U \): applied voltages (zero for passive circuits)
    - \( R \): resistance matrix
    - \( W \): poloidal beta and internal inductance \([\beta_{pol}, \ell_i]^T\)
    - \( Y \): including quantities of interest (gaps, magnetic signals, etc.)

- Grad-Shafranov constraint imposed numerically (for given geometry, circuit connections and materials, the only inputs to an MHD code are \( I \) and \( w \))
Perturbed equilibrium approach (3)

- **nonlinear form:**
  \[
  \frac{d\Psi}{dt} + RI = U \quad \text{(circuit equations)}
  \]
  \[
  [\Psi, Y]^T = \eta(I, W) \quad \text{(Grad-Shafranov constraint in symbolic form)}
  \]

- **linearized form (assuming \(I=I_0+i, \ U=U_0+u, \ W=W_0+w, \ Y=Y_0+y\))**
  \[
  L^* \frac{di}{dt} + R i = u - L_{E^*} \frac{dw}{dt} \quad \text{(\(L^* = \partial \Psi/\partial I, \ L_{E^*} = \partial \Psi/\partial W\))}
  \]
  \[
  y = C i + F w \quad \text{(\(C = \partial Y/\partial I, \ F = \partial Y/\partial W\))}
  \]

- **state-space form (assuming \(x=i, \ A=-(L^*)^{-1}R, \ B=(L^*)^{-1}, \ E=-(L^*)^{-1}L_{E^*}\))**
  \[
  \frac{dx}{dt} = Ax + Bu + Ew
  \]
  \[
  y = Ci + Fw
  \]

  - \(L^*\) is an inductance matrix modified by the presence of the plasma
  - linearization carried out using any MHD equilibrium code using incremental ratios (e.g., \(\Delta \Psi/\Delta I\)) or Jacobian matrix (readily available with Newton’s method)
  - state-space matrices strongly depending on the plasma configuration: time-varying in various phases of scenario and even discontinuous in time in limited-diverted transition
Perturbed equilibrium approach (4)

- **growth rate and stability margin**
  - same definitions (and problems) as rigid massless model:
    - growth rate: \( A x_u = \gamma x_u, \quad \gamma > 0 \)
    - stability margin: \( m_s = - (x_u^T L * x_u)/(x_u^T L x_u) \)
  - more realistic predictions than rigid massless model:
    - rigid model predictions inaccurate especially for triangular plasmas
      (more elongated inwards than outwards → non-rigid motion)

ITER start of burn configuration

- high \( \delta \)
- high \( b/a \)
- \( (I_p=15 \text{ MA}, \ I_\text{ii}=0.85, \ \beta_{\text{pol}}=0.65) \)
Perturbed equilibrium approach (5)

- Example of application to the JET tokamak

JET shot
#54283@64s 1.5 MA, I_t=0.9, \( \beta_{pol}=0.1 \)

open loop simulation of a VDE
(\( \gamma_{exp}=420 \text{ s}^{-1} \), \( \gamma_{sim} = 460 \text{ s}^{-1} \))
ZPDIP = VS controlled signal
Excitation of the unstable mode (1)

How is a vertical instability triggered?
By any disturbance exciting the unstable mode, e.g.
- an externally applied field
  - a radial field “kick” from an external circuit
- a change in the current density profile
  - an ELM (edge localized mode)
  - a soft disruption

ALL MODES IN PRINCIPLE EXCITED
- ELMs do not excite vertical instability of up-down symmetric configuration
- possible existence of neutral points also for unsymmetric configurations

ALL MODES STABLE EXCEPT ONE (THE GROWTH MODE)
Excitation of the unstable mode (2)

Modelling:
- external circuits included in the model
- ELMs modeled as equivalent jumps of $\beta_{pol}$ and $l_i$
  
  e.g., $\Delta w = [\Delta \beta_{pol} = -10\% \beta_{pol} \; \Delta l_i = +5\% \beta_{pol}]$

Quantitative estimation (useful for controller design):
- find initial conditions:
  
  \[
  \frac{dx}{dt} = Ax + Bu + E \frac{dw}{dt} \Rightarrow x(0^+) = E \Delta w
  \]
- remove all stable components from the initial conditions:
  
  \[
  x(0^+) = \alpha_u x_u + \sum_k \alpha_{sk} x_{sk}
  \]
- evaluate a physical parameter corresponding to the unstable mode at $0^+$, e.g. the vertical displacement $\delta z_{0u} \neq \delta z_0(0^+)$:
  
  \[
  \delta z_{0u} = \alpha_u c_z^T x_u \quad (c_z^T: \text{row of output matrix } C)
  \]
Diagnostics for vertical stabilization

- Measurable current moments (linear combinations of fluxes and fields: 
  \[ \sum_k z_k I_k \quad \text{and} \quad \sum_k I_k \]

- Vertical speed:
  \[ I_p \ddot{z}_p = \frac{d}{dt} \left( \sum_k z_k I_k \right) - z_p \dot{I}_p - \sum c \dot{I}_c \]

- Problems:
  - time derivatives noisy
  - non-measurable surrounded currents

Any signal might be used as controlled variable, provided that it is sensitive to the growth mode (which depends on the configuration) and is scarcely sensitive to ELMs, radial plasma motion and external currents.
Feedback stabilization (1)

PRINCIPLES

• open loop:

\[
L \dot{\delta I} + g \delta \dot{z}_p + R \delta I = \delta U
\]

\[
0 = g^T \delta I + F' \delta z_p
\]

\[
\delta U = k \delta \dot{z}_p
\]

• closed loop, e.g. with derivative action:

\[
L \dot{\delta I} + g \delta \dot{z}_p + R \delta I = \delta U
\]

\[
0 = g^T \delta I + F' \delta z_p
\]

\[
\delta U = \left(L - gg^T / F'\right) \dot{\delta I} + R \delta I = \delta U
\]

\[
\left[L + (k - g)g^T / F'\right] \dot{\delta I} + R \delta I = 0
\]

(character of closed loop $L^*$ may change)

• Various philosophies: PD, PID, bang-bang, adaptive, etc.
Feedback stabilization (2)

INSTALLED POWER REQUIRED

Superpose open loop (free) evolution and (forced) response to active voltage

- starting from:
  - connections of active and passive circuits
  - equilibrium configuration
  - applied disturbance (to get initial conditions)
  - maximum tolerable displacement

- apply a voltage step of given amplitude to the active circuit and find:
  - if it is able to stop the unstable motion (invert speed) before exceeding tolerable displacement
  - values of time, currents and displacement at speed inversion

- determine lower bounds for installed voltage and current

- take safety margins
Feedback stabilization (3)

AN INDICATION OF ACTIVE CIRCUIT PERFORMANCE

Bode Diagrams for the dBr/dt (radial field derivative) output channel in JET and ITER.
The input channel is the stabilizing voltage (VFRFA for JET, VS1 for ITER).
Both transfer functions can be approximated as 2nd order systems.
Experimental observations (1)

GROWTH RATE

Estimation of the growth rate in JET pulse #54839: exponential fitting of ZPDIP (the controlled variable)

- notice stable modes at the beginning of the VDE
- equilibrium value of ZPDIP = 0
- applied control voltage $V_{FRFA} = 0$
- techniques available also for nonzero equilibrium conditions and applied voltages
Experimental observations (2)

ELMs

JET pulse #60682

\[ I_p \approx 2 \text{ MA} \]
\[ \beta \approx 0.75 \]
\[ l_1 \approx 0.85 \]
\[ W_{\text{DIA}} \approx 4.4\text{MJ} \]
Experimental observations (3)

ELMs with VDEs avoided

complex phenomenon, BUT equivalent model for VS

\[ \Delta \beta_p \approx -10\% \beta_{pol} \quad \Delta I_i \approx +5\% \beta_{pol} \quad \Delta I_p \approx 0 \]

JET pulse #60682

- \[ \Delta I_p \approx 0 \] (then small variation)
- \[ \Delta z_p \] jump (>0 or <0) (then recovery)
- \[ \Delta r_p \] jump (<0) (due to loss of pressure energy)
- \[ \Delta \beta_{pol} \] jump (<0) (loss of pressure energy)
- \[ \Delta W_{\text{DIA}} \leq 10\% W_{\text{DIA}} \]

Do we believe measurements on short time scale
(Rogowski coils also surround passive currents) ???
Experimental observations (4)

A GIANT ELM FOLLOWED BY A VDE

JET pulse #52314
Current limit reached:
Voltage switched off
Plasma in open loop (VDE)

In present JET operations VFRFA is switched off for hundreds of microseconds by the time an ELM is detected (via H-ALPHA radiation signal)
The JET stabilization system including power supply, coil connections, control algorithm and detection system is currently being enhanced

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Experimental observations (5)

FINAL PHASE OF VDE: HALO CURRENTS & CURRENT QUENCH (a)

The plasma hits the wall after a ≈12 cm displacement:
• diverted-limited transition
• plasma response discontinuity
  • plasma speed jump
  • growth rate jump
• rise of halo currents:
  • vertical speed slowing down
  • plasma temperature decreasing
  • impurities increasing
  • higher plasma resistivity
  • time constant of poloidal eddy currents
• plasma disruption:
  • temperature quench
  • plasma current quench

JET pulse #54283 (γ=410 s⁻¹)
Experimental observations (6)

FINAL PHASE OF VDE: HALO CURRENTS & CURRENT QUENCH (b)

The plasma hits the wall after a $\approx 12$ cm displacement ($t \approx 64.015$ s):
- diverted-limited transition
- plasma response discontinuity
- rise of halo currents (at $t \approx 64.020$ s):
  - vertical speed slowing down
  - plasma temperature decreasing
  - impurities increasing
  - higher plasma resistivity
  - time constant of poloidal eddy currents
- plasma disruption ($64.025 < t < 64.045$ s):
  - temperature quench
  - plasma current quench

$ZPDIP \approx d(zpIp)/dt$

JET pulse #54283 ($\gamma = 410$ s$^{-1}$)

- Halo currents flow partly in the plasma, partly in the wall
- Halo measured by TF unbalance inside the vessel or currents flowing through FW tiles
Experimental observations (7)

Consequences of VDEs and AVDEs

- **VDEs are more dangerous than other disruptions:**
  - Large vertical forces (due to both eddy and halo currents)
  - Force concentration in upper (or lower) part of the structures
  - Halo currents: large thermal loads on the wall

- **Asymmetric VDEs are even more dangerous:**
  - Not yet clearly understood
  - Net horizontal (sideways, not radial) forces due to combined tilting and horizontal displacement (coming from the interaction with the toroidal field) superposed to vertical motion
  - Force concentration in some toroidal sectors (toroidal peaking factors to be considered)
  - Halo current concentration: large thermal loads on the wall (empirical toroidal peaking factors)
References (1)

MHD Stability


Vertical instability


Rigid displacement model

References (2)

Perturbed equilibrium model


Feedback stabilization

References (3)

Eddy current treatment


ELMs


VDEs, AVDEs and halo currents