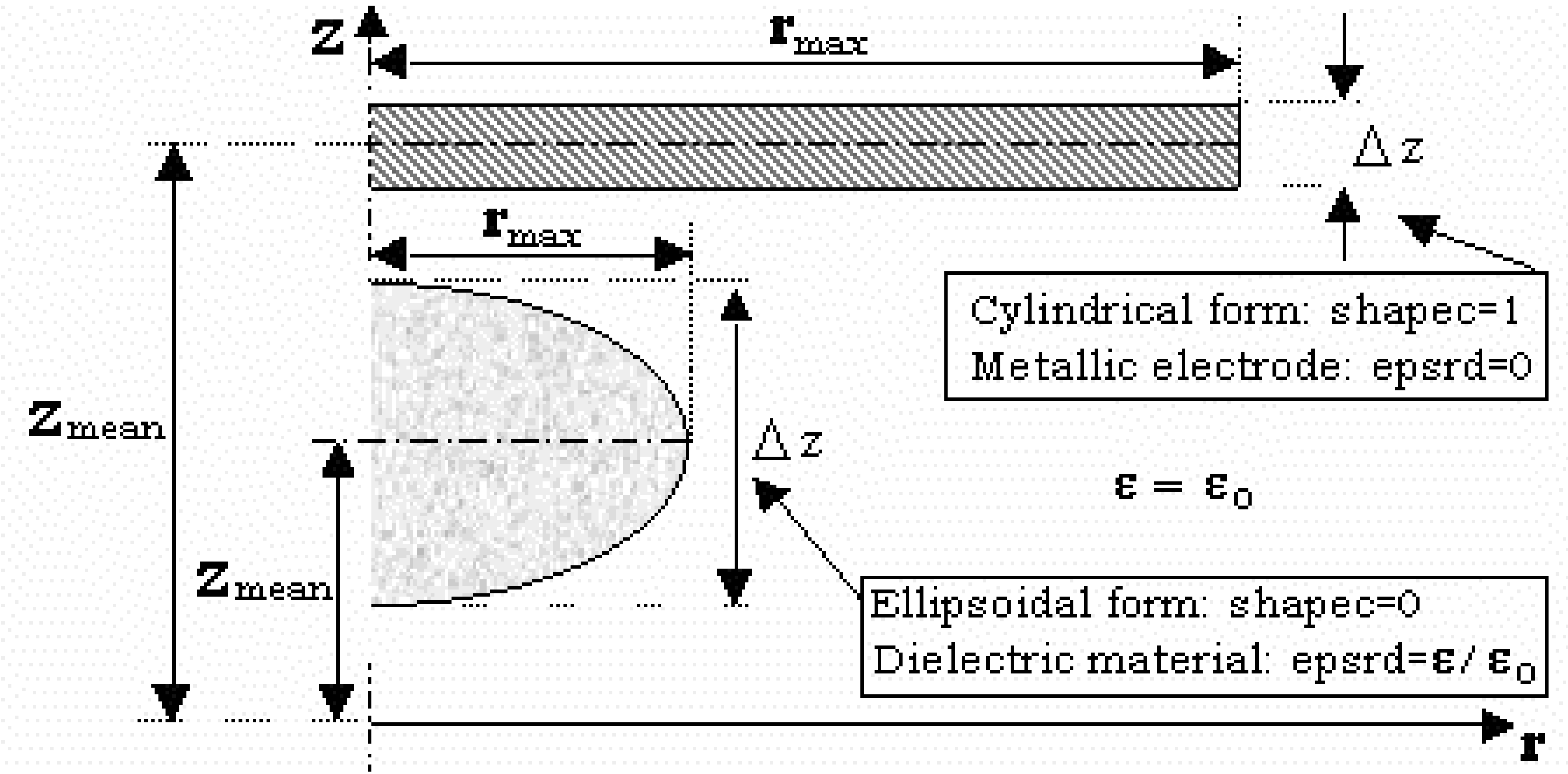
Lezione n. 7

Un esempio di applicazione del metodo degli elementi finiti: calcolo della capacità di un condensatore piano

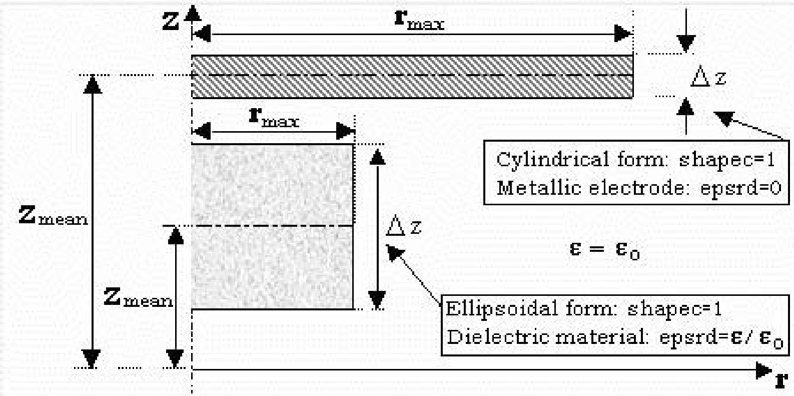


Lez_7bis_Capacitance_matrix_adv_it.ppt

from

http://nettuno.unina.it/guardians/resources.html

Condensatore piano: INPUT



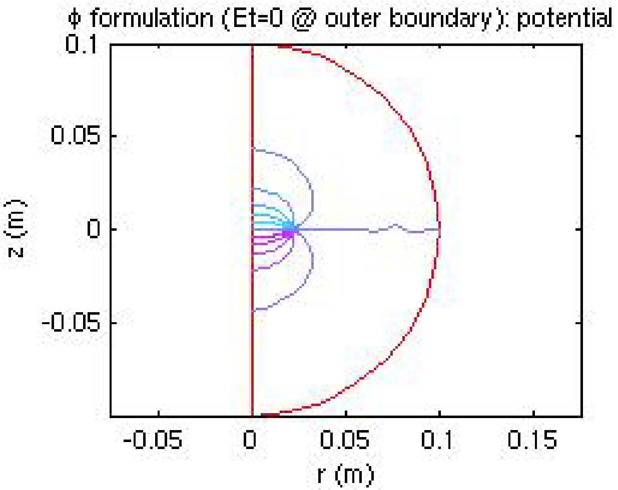
parametri

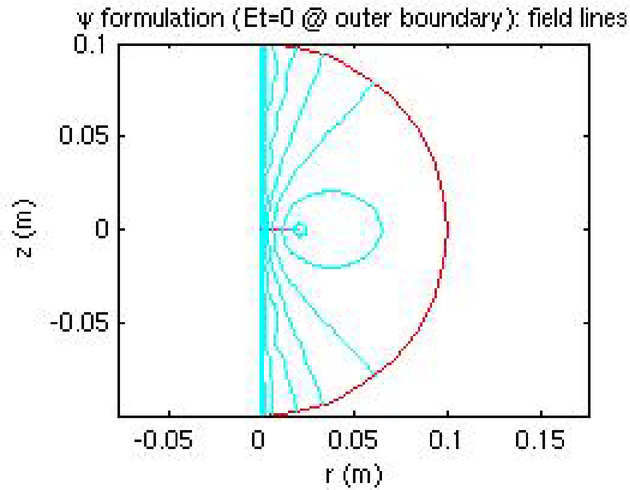
mesh definition [0 normal, 3 very high] 0

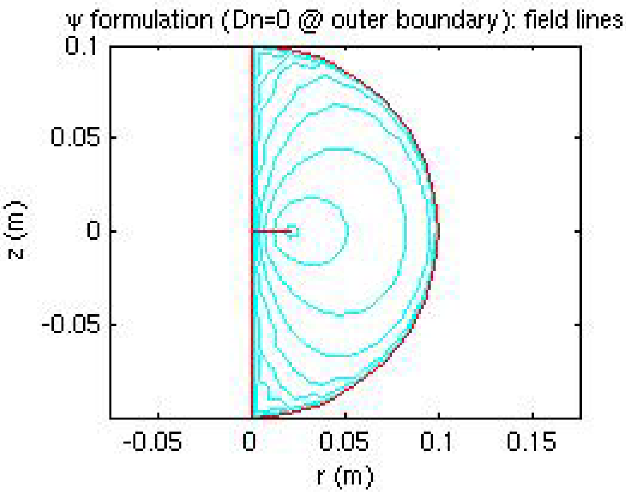
	rmax	dz	zmean		
Conductor 1	20e-3	1.e-4	-2e-4	1	0
Conductor 2	20e-3	1.e-4	2e-4	1	0
Conductor 3	20e-3	1.e-4	0	1	2

Start simulation

Condensatore piano: OUTPUT



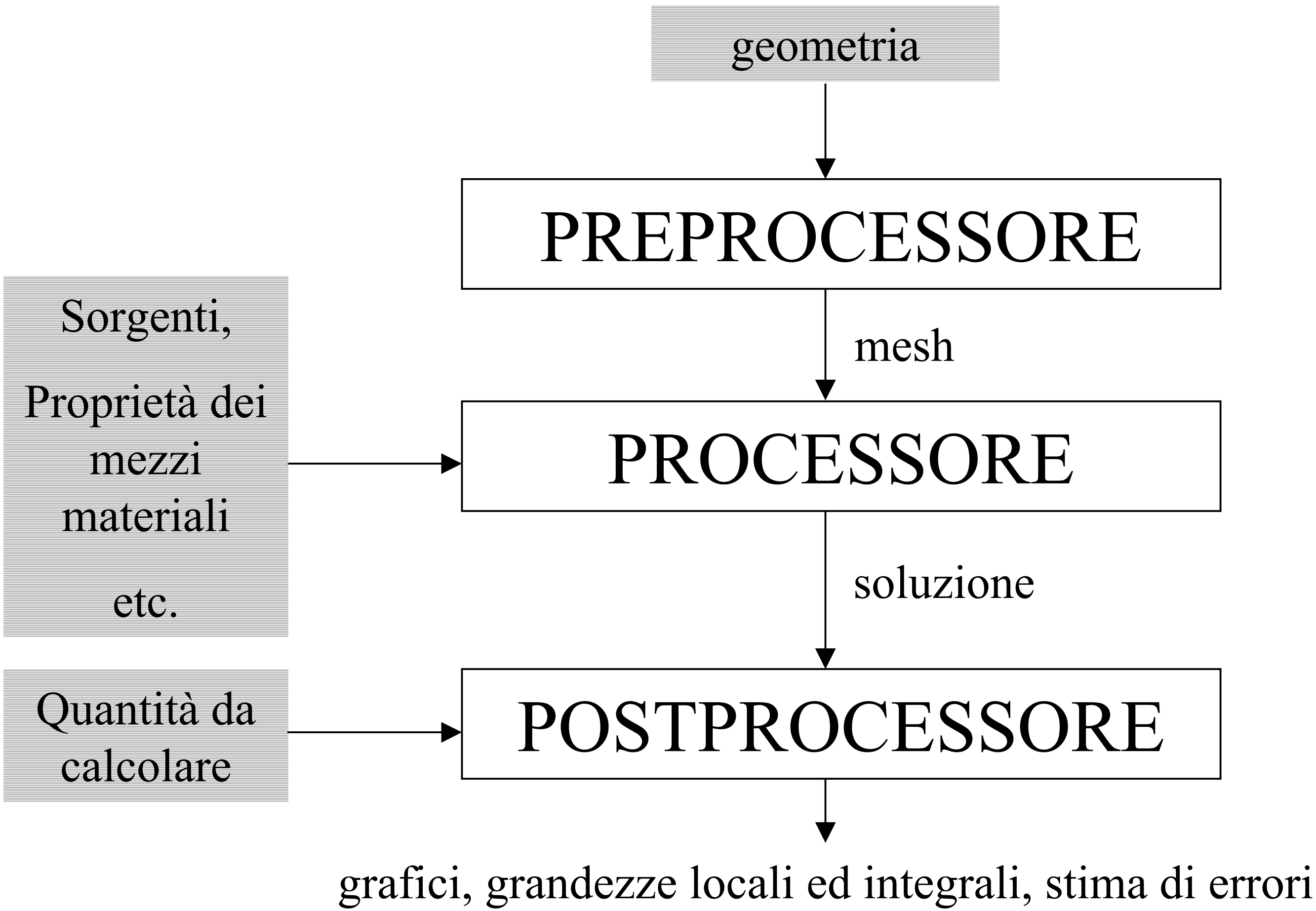




Capacitance estimates:
ϕ formulation (Et=0 @ outer boundary): C = 4.601e-11 Farad
ψ formulation (Et=0 @ outer boundary): C = 4.582e-11 Farad
ψ formulation (Dn=0 @ outer boundary): C = 4.5816e-11 Farad
approximate analytic formula: C = 4.4506e-11 Farad

Lower and upper bounds:
4.5816e-11 F <= Capacitance <= 4.601e-11 F

Struttura di un codice FEM



```
function [fig_handler,C_phi, C_psi, C_psi0, C_an]=axisymm_c(NR,
        data_c)

% Program axisymm_c - by R. Albanese
% for the calculation of the capacitance of a plane capacitor
% in the 2D axisymmetric case
%
% the two electrodes and the dielectric regions are coaxial disks
%
% the calculation is carried out both in terms of scalar electric potential
% and in terms of electric flux via finite element method
%
% the regularity conditions at infinity are approximated by including
% the whole system inside a large conducting hollow sphere
%


% set of parameters decribing coaxial electrodes and/or dielectric
        regions
% shapec: =1 -> cylindrical shape: 0<=r<=rmax, zmean-
        dz/2<=z<=zmean+dz/2
% =0 -> ellipsoidal shape: (r/rmax)^2+((z-zmean)/(dz/2)^2<=1
% epsrd: >0 -> dielectric region having relative dielectric constant epsrd
% =0 -> conductive region (electrode)
% rmax(1:N,1): max radius
% dz(1:N,1): vertical thickness at r=0
% zmean(1:N,1): vertical position of the center
% shapec(1:N,1): 1 if cylindrical, 0 if ellipsoidal
% epsrd(1:N,1): relative dielectric constant if >0, 0 if conductive region
        (electrode)
%
% NOTICE: all the results are consistent if:
% 1) all regions are cylindrical
% 2) the number of conductors is two (the first two regions)
% otherwise a warning message will be displayed

% INPUT parameters
% data_c(1:N, 1:5): = [ rmax dz zmean shapec epsrd]
% NR: = No. of mesh refinements [min=0 normal, max=3 very high
        definition]

% OUTPUT parameters
% fig_handler: handle of the figure reporting the results
% C_phi: capacitance calculated in terms of scalar electric potential
% C_psi: capacitance calculated in terms of electric flux with Et=0 at
        outer boundary
% C_psi0: capacitance calculated in terms of electric flux with Dn=0 at
        outer boundary
% C_an: approximate analytical value of the capacitance

rmax=data_c(:,1);
dz=data_c(:,2);
zmean=data_c(:,3);
shapec=data_c(:,4);
epsrd=data_c(:,5);

% PRODUCE MESH & GEOMETRY DESCRIPTION
[p,e,t,gd,sf,ns,label]=capacitors_mesh(rmax, dz, zmean, shapec, epsrd);

% GET BOUNDARY LABELS
[boundreg]=findbl(gd,sf,ns,epsrd,label);

% GET SUBDOMAIN LABELS
[meshreg,meshreg0]=findml(gd,sf,ns,epsrd,label);

% PRODUCE ADDITIONAL GEOMETRY DESCRIPTION
[g,bt]=decsg(gd,sf,ns);

% REFINE MESH NR TIMES ( 0 <= NR <= 3)
for KR=1:NR,[p,e,t,dum]=refinemesh(g,p,e,t,zeros(size(p,2),1));
        p=jigglemesh(p,e,t); end
```

```
% FIND COMPLEMENTARY SOLUTIONS (NOTE: PHI & PSI ARE
        COMPLEMENTARY IN THE TRUNCATED DOMAIN
% PHI & PSI0 ARE COMPLEMENTARY IN THE UNBOUNDED DOMAIN)
[phi,Wphi]=finds(p,e,t,epsrd,meshreg,boundreg);
[psi,Wpsi]=finds_psi(p,e,t,epsrd,meshreg,boundreg,1);
[psi0,Wpsi0]=finds_psi(p,e,t,epsrd,meshreg,boundreg,0);

% GET CAPACITANCE MATRICES
Cphi=2*Wphi ;
Cpsi=inv(2*Wpsi) ;
Cpsi0=inv(2*Wpsi0);

% CALCULATE CAPACITANCE WITH PHI FORMULATION
% capacitor field: cond. #1 charged with +1 Coulomb, cond. #2 charged with -1
        Coulomb
V = inv(Cphi)*[1;-1];
phi_c=phi*V;
W_c=0.5*V'*Cphi*V;
C_phi=1/(2*W_c) ;

% CALCULATE CAPACITANCE WITH PSI FORMULATION
% capacitor field: cond. #1 charged with +1 Coulomb, cond. #2 charged with -1
        Coulomb
Q = [1;-1];
psi_c=psi*Q;
Cpsi=inv(2*Wpsi);
W_c_psi=0.5*Q'*inv(Cpsi)*Q;
C_psi=1/(2*W_c_psi) ;

% CALCULATE CAPACITANCE WITH PSI0 FORMULATION
% capacitor field: cond. #1 charged with +1 Coulomb, cond. #2 charged with -1
        Coulomb
Q = [1;-1];
psi0_c=psi0*Q;
Cpsi0=inv(2*Wpsi0);
W_c_psi0=0.5*Q'*inv(Cpsi0)*Q;
C_psi0=1/(2*W_c_psi0) ;

% CALCULATE APPROXIMATE ANALYTIC VALUE OF THE
        CAPACITANCE

rmin=min(rmax); S=pi*rmin^2; eps0=8.8542e-12;
zcsorted=sort([zmean(1)-dz(1)/2;zmean(1)+dz(1)/2;zmean(2)-
        dz(2)/2;zmean(2)+dz(2)/2]);
zmin=zcsorted(2); zmax=zcsorted(3);
[dum,isorted]=sort(zmean); zmeans=zmean(isorted); dzs=dz(isorted);
        epsrds=epsrd(isorted);
Cz=[];
for k=1:length(zmeans);
zmink=zmeans(k)-dzs(k)/2;
zmaxk=zmeans(k)+dzs(k)/2;
if zmink>zmin
Cz=[Cz; eps0*S/(zmink-zmin)];
if zmaxk Cz=[Cz; eps0*epsrds(k)*S/(zmaxk-zmink)];
zmin=zmaxk;
end
end
end
C_an=1/sum(1./Cz);

%DISPLAY RESULTS
close all
h=figure(1);

% EQUIPOTENTIAL LINES (truncation with Et=0)
subplot(2,2,1)
pdemesh(p,e,[]), axis equal, hold on, pdecont(p,t,phi_c), hold on,
        pdecont(p,t,phi_c,logspace(-10,0,20))
title([' \phi formulation (Et=0 @ outer boundary): potential '])
xlabel(' r (m)')
ylabel(' z (m)')

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