Metodi e tecniche di ottimizzazione innovative per applicazioni elettromagnetiche

Ottimizzazione multiobiettivo

Parte 2 Tecniche di scalarizzazione

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Aggregation tecniques(1/2)

- Aggregation or scalarization techniques follow a different approach and have been extensively used in the past
- they mingle together the two steps of VOP solution because they can possibly find a particular configuration on the Pareto front
- Different approaches have been devised and they will be briefly outlined:
  - weighting of objectives
  - constrained approach
  - goal programming
  - fuzzy combination of objectives

Aggregation tecniques(2/2)

- Also aggregation techniques have not a univocal implementation so that the methods presented have been interpreted by researchers in different ways
- Some critical points are shared by all the aggregation techniques:
  - objectives do not share the same physical dimensions (incommensurability), before aggregation they have to be normalized
  - objectives do not have the same variation range in the problem domain
  - the analysis of only few points on the Pareto front can be misleading
  - several scalar optimization runs must be performed

Weighting of objectives (1/3)

- This method is the oldest multiobjective technique used
- The method finds its theoretical basis in the Kuhn-Tucker conditions for noninferiority
- Scalar objective function is defined as:

$$O(\mathbf{X}) = \sum_{k=1}^{M} w_k O_k(\mathbf{X}) w_k \geq 0., k = 1, ..., M$$

- weights w_k act as:
  - normalization constants
  - preference indicators giving more importance to an objective wrt to another

Weighting of objectives (2/3)

- Following the definition of the new objective function it is easy to see that the extremum point lies on the Pareto front:

$$O(\mathbf{X}) \Rightarrow \nabla O(\mathbf{X}) = 0 \Rightarrow \sum_{k=1}^{M} w_k \nabla O_k(\mathbf{X}) = 0$$

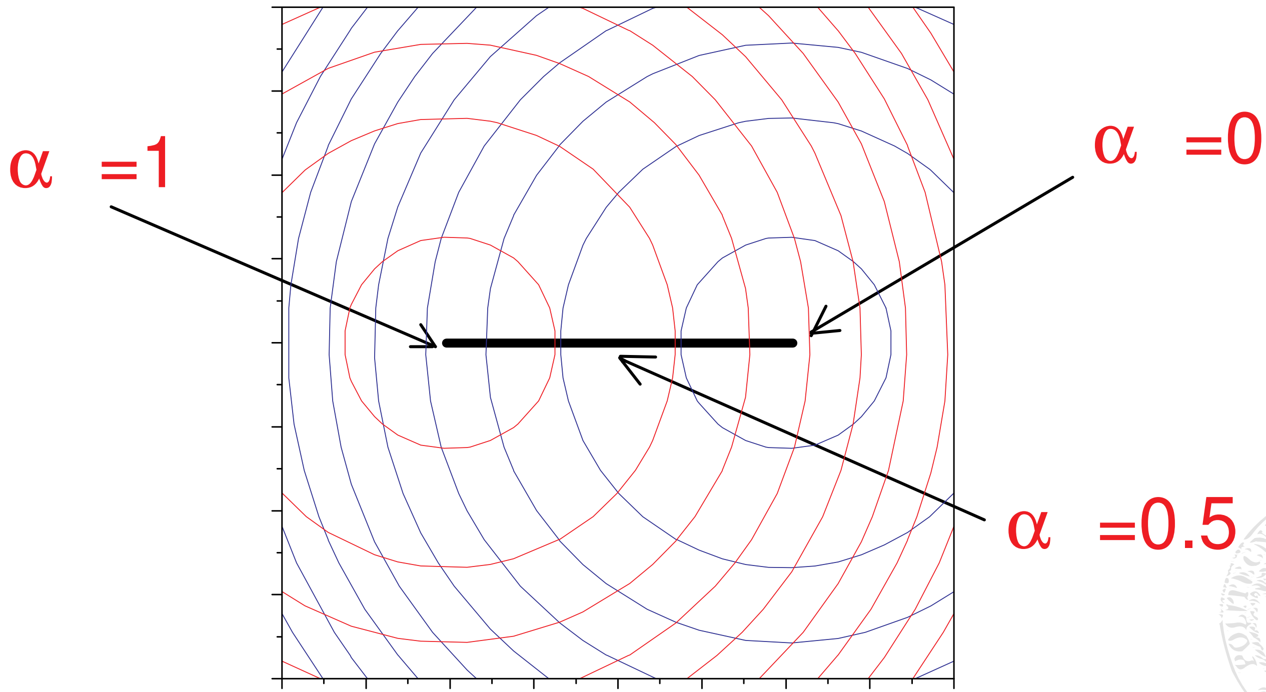
- this, under the hypothesis specified by K-T conditions, allows to say that the point is on the Pareto front
- since weights w_k have been selected "a priori" this choice will define which particular point on the Pareto front will be hit
- changing the set of w_k values allows one to explore the noninferior set at the cost of one optimization run per point

Weighting of objectives (3/3)

- Using again the two objectives example (same physical dimensions)

$$O_1(x,y) = 1 - (x-1)^2 - y^2 \quad O_2(x,y) = 1 - (x+1)^2 - y^2$$

$$O(x,y) = \alpha * O_1(x,y) + (1-\alpha) * O_2(x,y)$$



Payoff table

- As it was pointed out previously, extrema of Pareto front can be obtained by setting all the weights to 0 except one set to 1
- A table containing the M combinations can give some useful knowledge about the problem and variation range of the objectives

w_1	w_2	O_1	O_2
1	0	1	-12
0	1	-12	1

- The analysis of the table allows one to have a measure of the possible tradeoff among different objectives

Constrained approach (1/2)

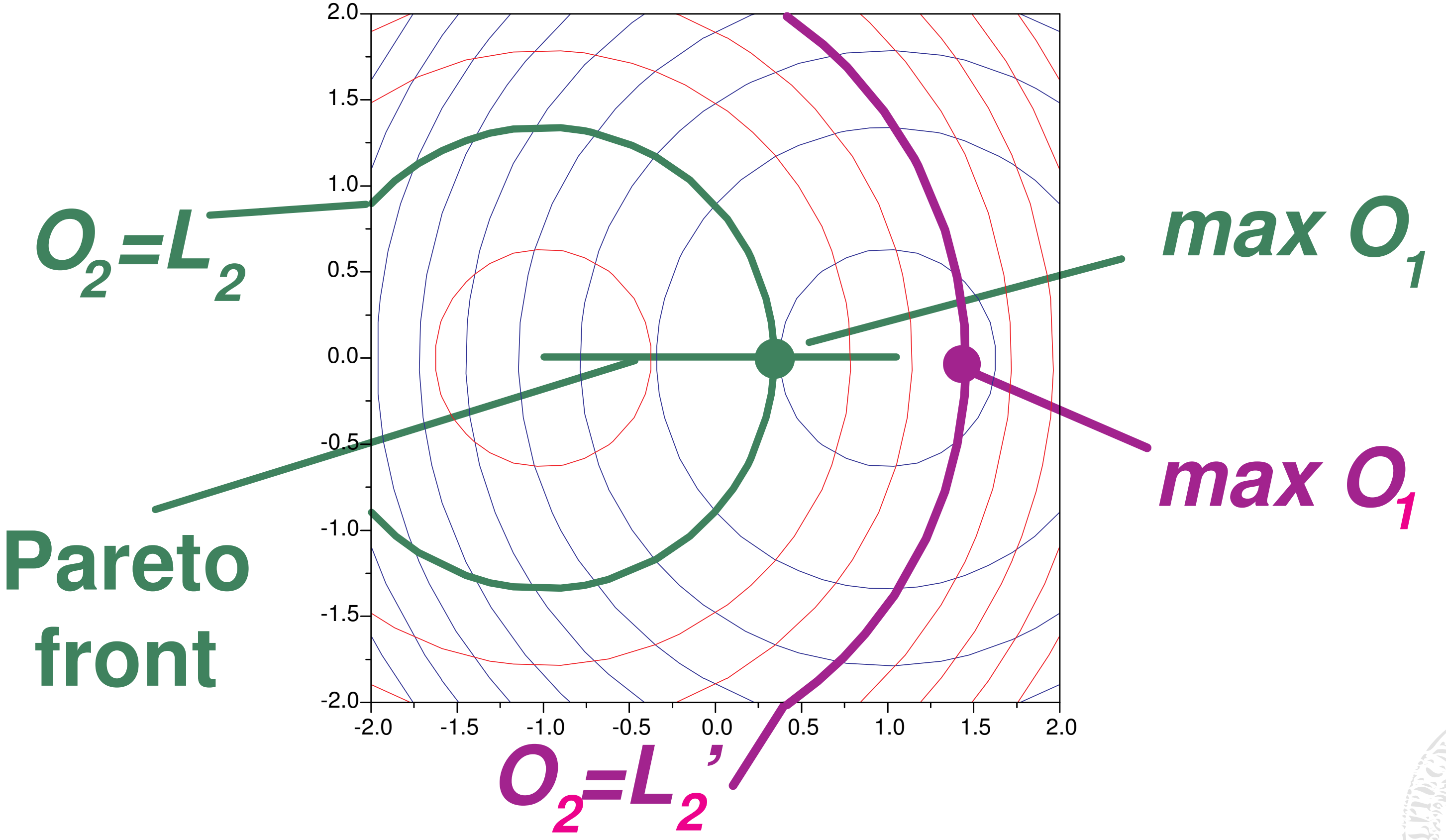
- Another approach which converts the VOP in a scalar optimization problem requires to change the formulation:

$$\text{maximize } O_i(\mathbf{X})$$
$$\text{subject to } O_j(\mathbf{X}) = L_j \quad j = 1, ..., M, \; j \neq i$$

- the objective transformed in constraints are fixed to a given value L_j which has to be stated by the user
- obviously this approach requires to have an optimization procedure able to handle constrained problem
- changing the set of L_j values change the optimal point, if the values of L_j are suitably selected, the optimal point lies on the Pareto front

Constrained approach (2/2)

- Maximization of objective O_1 subject to $O_2 = L_2$



Goal Programming (1/4)

- The Goal Programming method looks for a VOP solution by a prior specification of a "best" solution
- This *best* solution can be in general unfeasible as the one that maximizes all the objectives at the same time, in some implementation this configuration is called in fact utopia $\{G_1, G_2, ..., G_M\}$

$$minimize \quad d(\mathbf{X}) = \sum_{k=1}^{M} w_k \mid G_k - O_k(\mathbf{X}) \mid$$

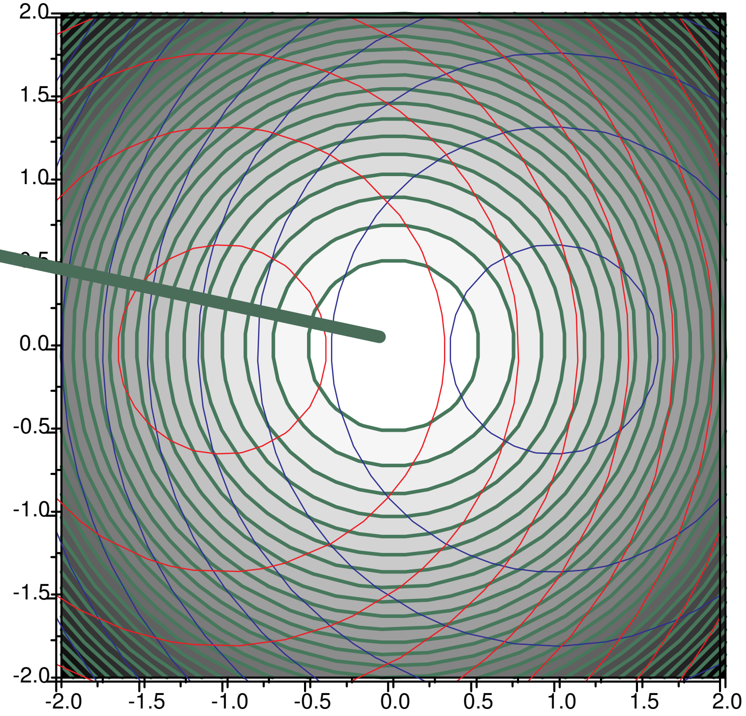
- weights w_k are needed because in general O_k are incommensurable

Goal Programming (2/4)

- the optimal point found depends on the set of normalization constants w_k and on the set of goals G_k
- Several implementations of the goal programming have been presented changing the distance definition (euclidean, max etc.)
- In general the optimal point found does not belong to the Pareto front unless the goal point is not on the Pareto front or it is unfeasible
- Again an exploration of the Pareto front needs several optimization runs changing the goal set.

Goal Programming (3/4)

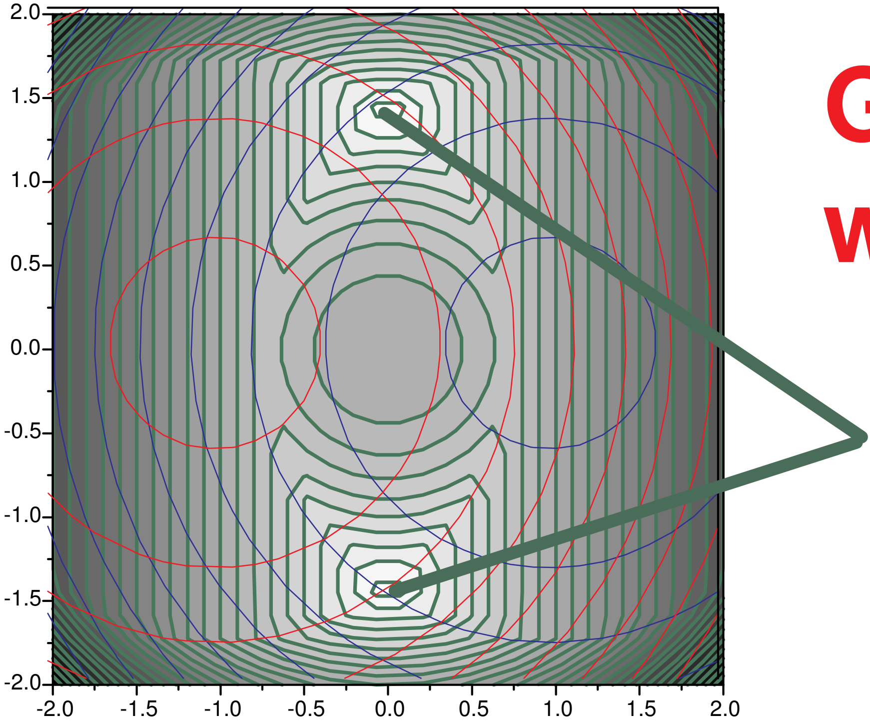
minimum on
Pareto front



$G_1=2$, $G_2=2$ unfeasible

$w_1=w_2=1$

Goal Programming (4/4)



G_1=-2, G_2=-2 dominated

w_1=w_2=1

minima out of Pareto front

Fuzzy logic (1/3)

- Among aggregation techniques, Fuzzy combination of objectives has become largely used in the last ten years
- Fuzzy Logic *(FL)* was originally proposed by L.A. Zadeh in 1965 for control applications in the area of multiattribute decision making.
- The new logic was proposed to overcome some problems created by "crisp" decision making, using instead some tools able to include an uncertainty level
- One of the most powerful issues of *FL* is the natural way of translating a sentence in a numerical information
- The multi-level logic allowed to define a series of logical operators similar to the ones of classical logic (AND, OR, NOT etc.)

Fuzzy logic (2/3)

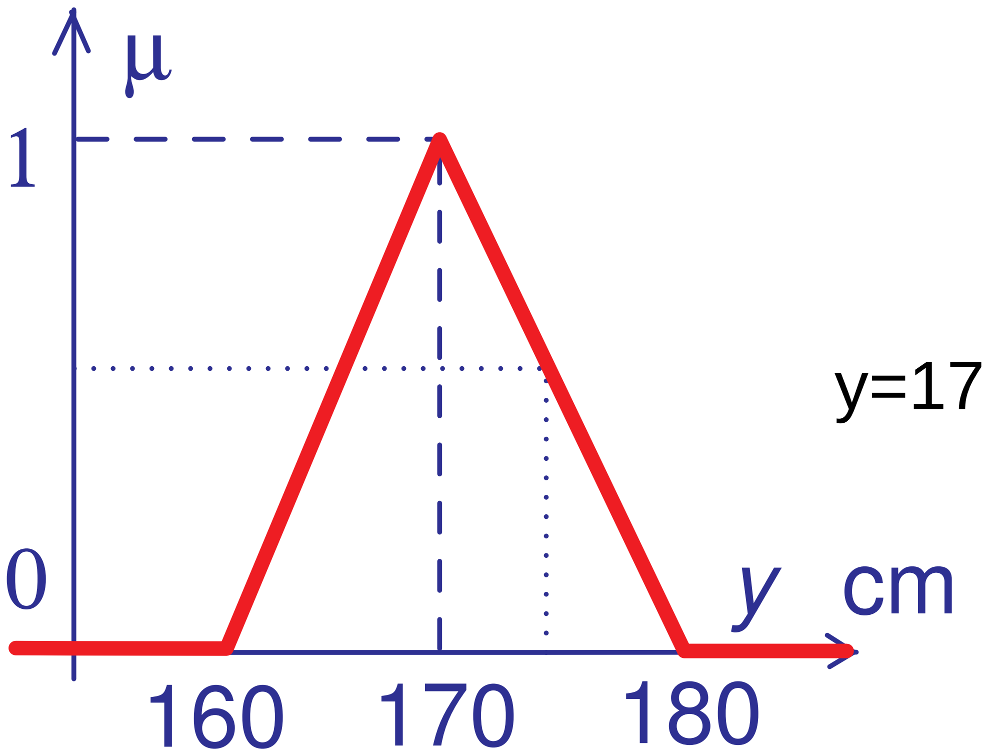
- Defining a numerical continuous truth level ranging from 0=false to 1=true, *FL* allows one to state the degree of truth of one proposition by means of a function:

$$\mu_A : Y \longrightarrow [0, 1]$$

where μ is called membership function and Y is its domain

Fuzzy logic (3/3)

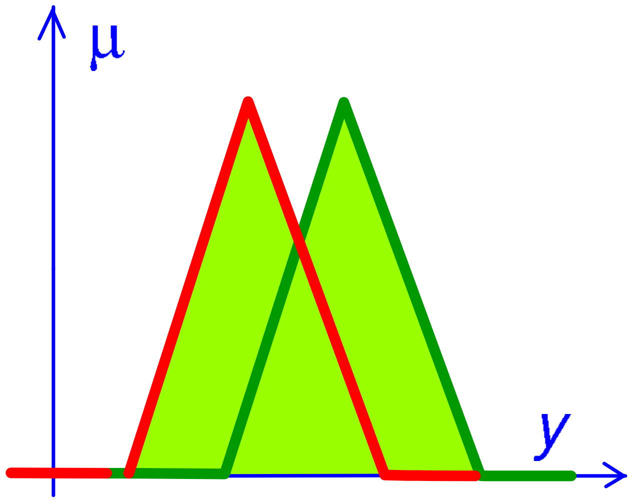
- Membership functions are very useful in defining the degree of truth of an uncertain statement the degree of belonging of a man to the "set of average height man"



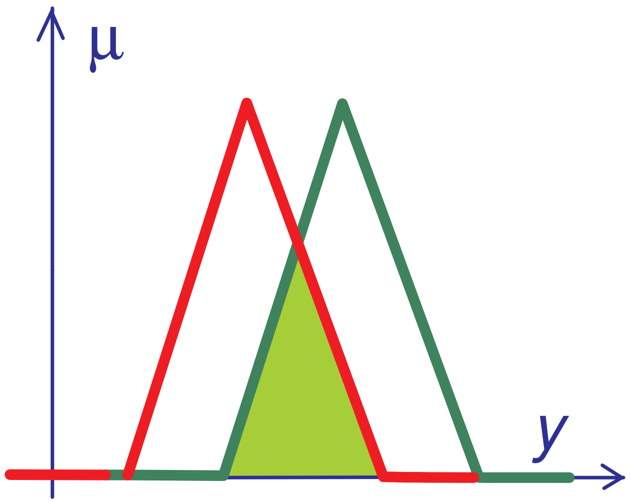
y=175 cm $\longrightarrow \mu = 0.5$

Fuzzy operators

- Logical operators can be defined on these new functions



$$\mu_A.OR.\mu_B$$
$$\max_{y \in Y}\{\mu_A(y), \mu_B(y)\}$$

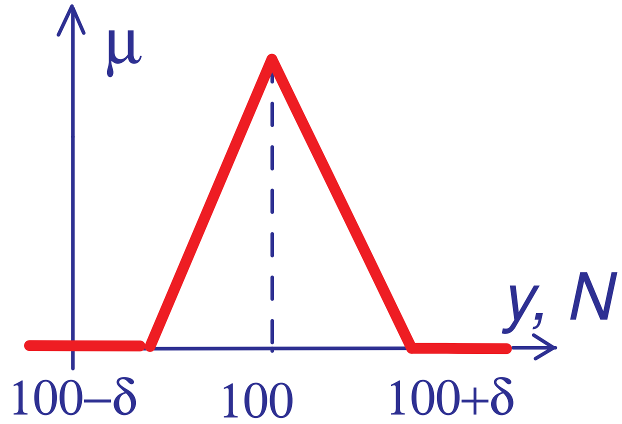


$$\mu_A.AND.\mu_B$$
$$\min_{y \in Y}\{\mu_A(y), \mu_B(y)\}$$

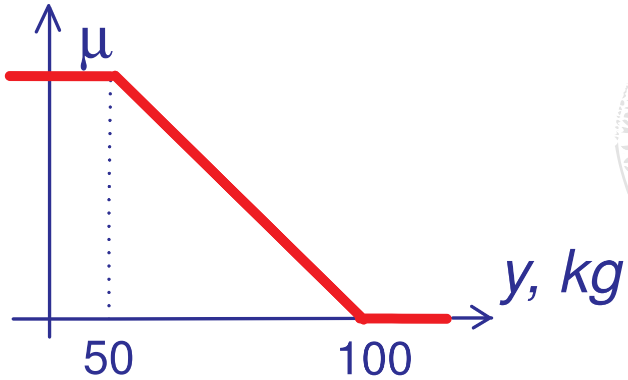
Fuzzy examples

- A degree of satisfaction of a certain objective can be expressed by means of a suitable membership function

Force must be around 100N and δ represents the maximum distance accepted from 100N



Weight must be as minimum as possible: over 100 kg is unacceptable, any value under 50 kg is accepted



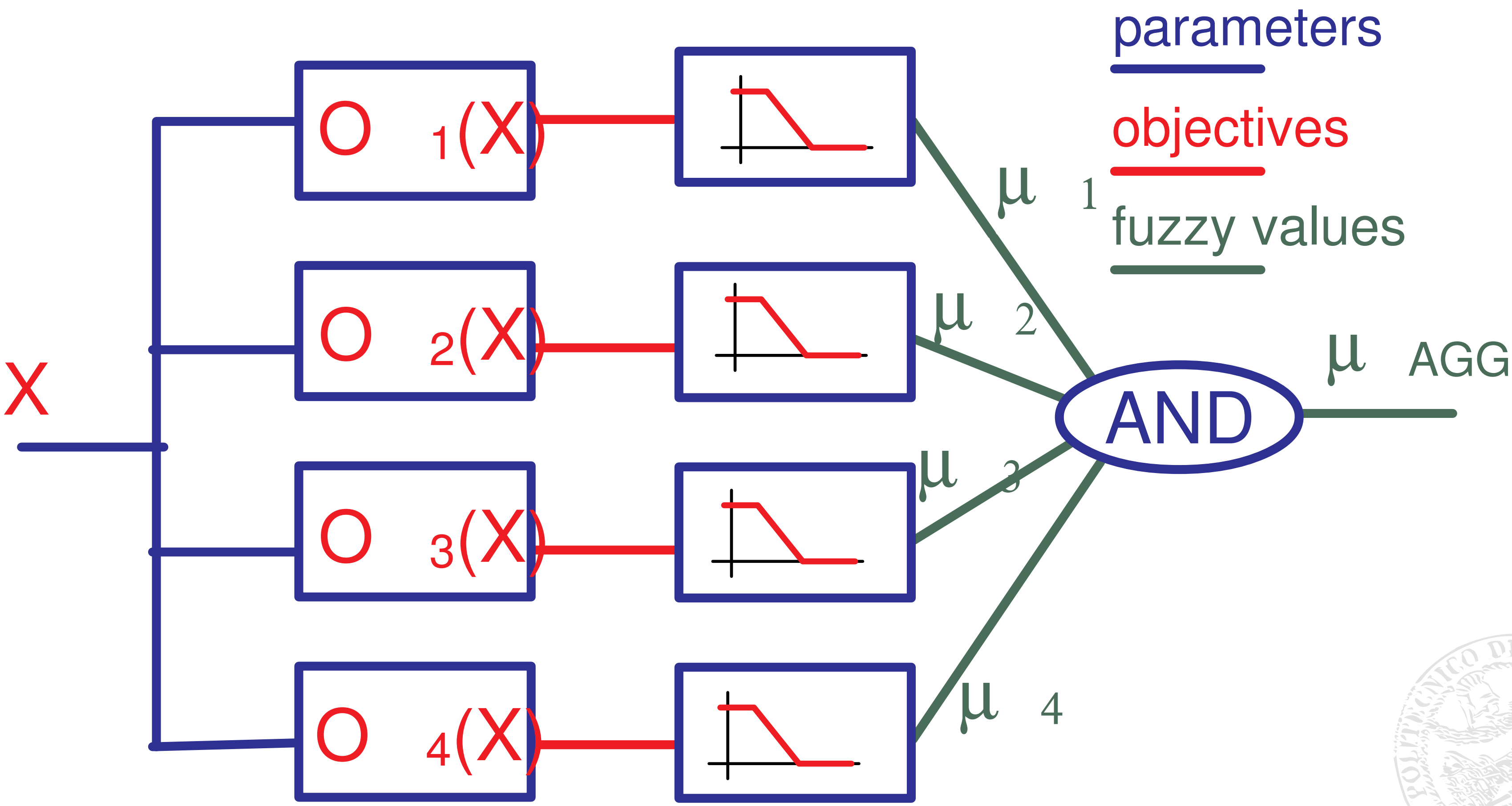
Fuzzy combination (1/)

- Membership functions perform a natural normalization of any quantity in a logical level, this allows to compare and combine different degrees of satisfaction in a global one
- The degree of acceptance of each configuration is defined as the intersection of all the single membership functions.
- The logical intersection operator is the AND one which in FL is interpreted by the minimum of all membership functions
- optimization $\Rightarrow$ maximizes the global degree of satisfaction:

$$maximize\ min\{\mu_j(O_j(\mathbf{X}))\} \quad j = 1, ..., M$$

Fuzzy combination (2)

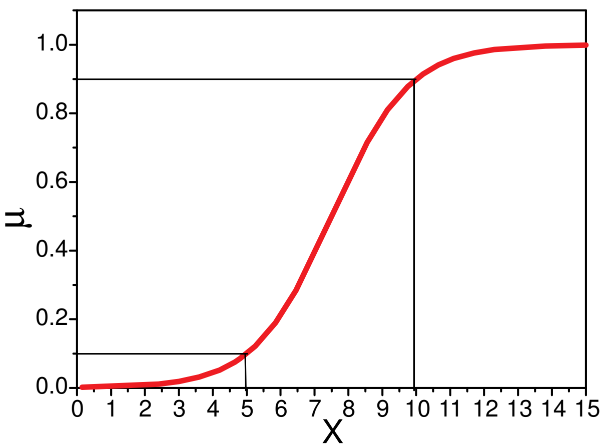
A flowchart of the fuzzy aggregation becomes:



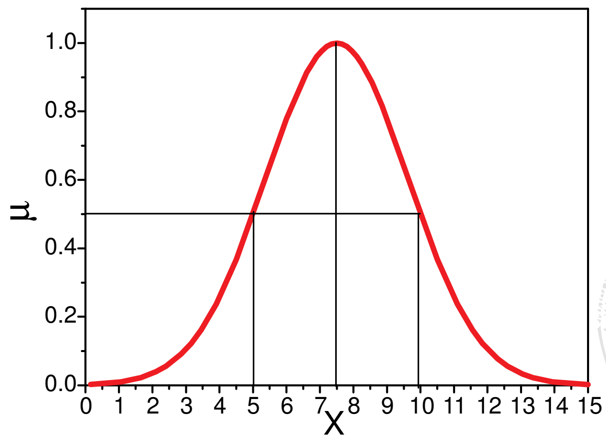
definition of μ

- Piecewise linear membership functions are simple to define but can give rise to problems for instance when a flat 0 or 1 value are obtained in a certain region
- to avoid this problem analytical μ can be defined using gaussian or sigmoidal functions

$$x = 5 \rightarrow \mu = 0.1$$
$$x = 10 \rightarrow \mu = 0.9$$



$$x = 7.5 \rightarrow \mu = 1.0$$
$$x = 5, 10 \rightarrow \mu = 0.5$$



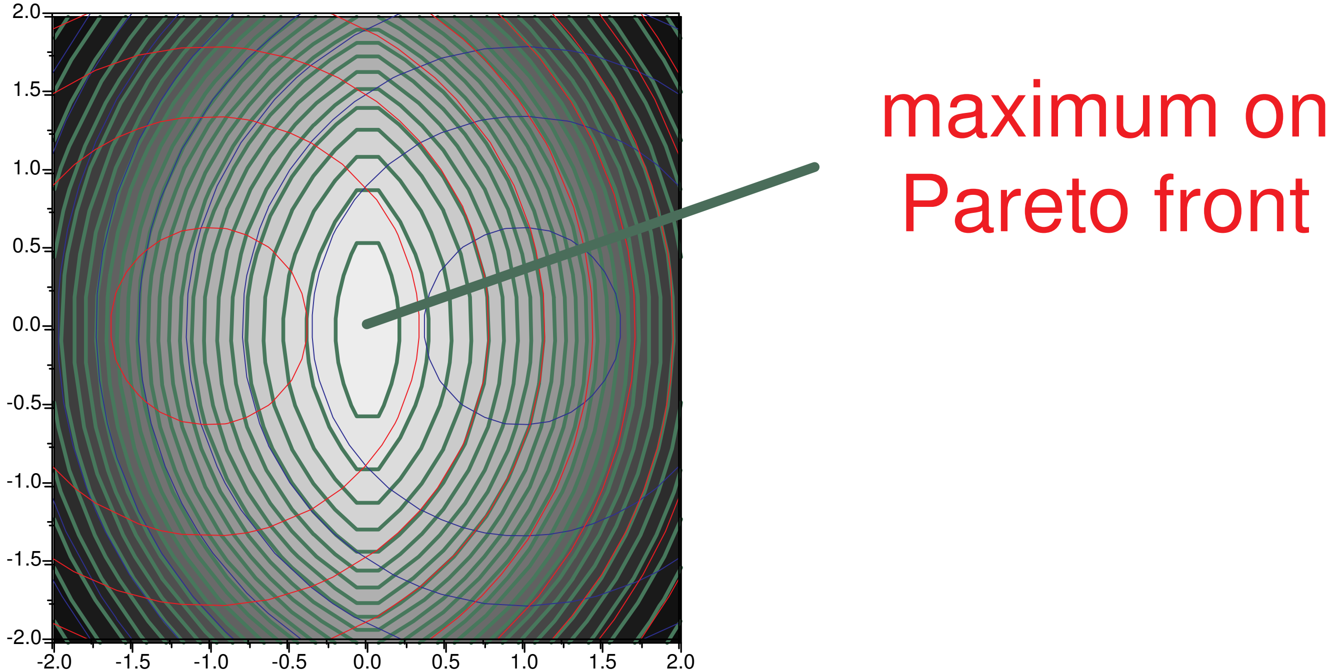
Pareto fuzzy approach

- Membership functions perform a mapping from objective space to "logical" space. Also in this space Pareto optimality can be defined.
- A configuration X_μ is fuzzy-Pareto optimal if there is no way of improving a single membership function without degradating any of the other
- Obviously, the solution of the fuzzy VOP lies on the Pareto front only if the maximum satisfaction levels specified in the m are higher than the ones of dominated solutions
- An exploration of the fuzzy-Pareto front can be performed by changing the limits of the membership functions

Fuzzy aggregation

- Defining two identical sigmoidal membership fuctions with the following parameters:

$$x = 1.5 \longrightarrow \mu = 0.9 \; x = -10 \longrightarrow \mu = 0.1$$



Fuzzy iterative refinement (1/2)

- The use of min (fuzzy AND) operator does not take into account the values of all membership functions in the global fuzzy indicator
- Only the objectives at the minimum value are taken into account in the optimization process, thus there is no information about other objectives.
- Configurations X_1 and X_2 are different but leads to the same global fuzzy indicator

$$\min\{\mu_A(X_1) = 0.25, \mu_B(X_1) = 0.25, \mu_C(X_1) = 0.5\} = 0.25$$
$$\min\{\mu_A(X_2) = 0.25, \mu_B(X_2) = 0.5, \mu_C(X_2) = 0.25\} = 0.25$$

- Obviously X_2 is better than X_1 but they give rise to the same global indicator

Fuzzy iterative refinement (2/2)

- In order to avoid situations like the previous one, a local refinement can be performed around an optimal point X_{opt}:

$$\mu(O_k(X_{opt})) = \overline{\mu_k}$$

- a new optimization can be defined as:

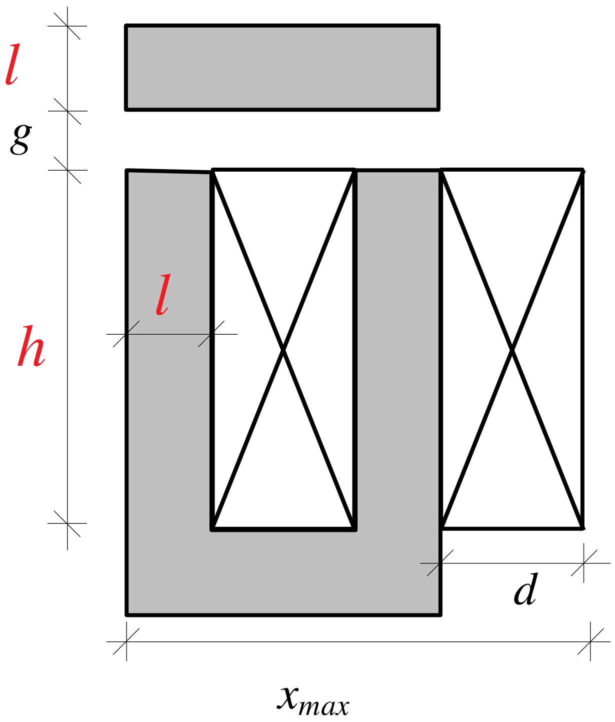
$$\text{maximize } \sigma_k(X)$$

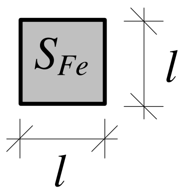
$$\{\sigma_k(X))\} = \left\{ \begin{array}{c} \mu_k(O_k(X)) - \overline{\mu_k},\ if\ \mu_k(O_k(X)) > \overline{\mu_k} \\ 0\ \ if\ \mu_k(O_k(X)) \leq \overline{\mu_k} \end{array} \right\} = 0$$

- obviously if X_{opt} is a Pareto optimal solution the second optimization is not able to find any better solution, otherwise increments in objectives with μ values greater than the minimum can be obtained

VOP example (1/2)

- A simple example of a VOP problem can be obtained by the optimization of an electromechanical actuator
- Electromechanical actuators are very simple to analyse but they are a good test on the optimization point of view

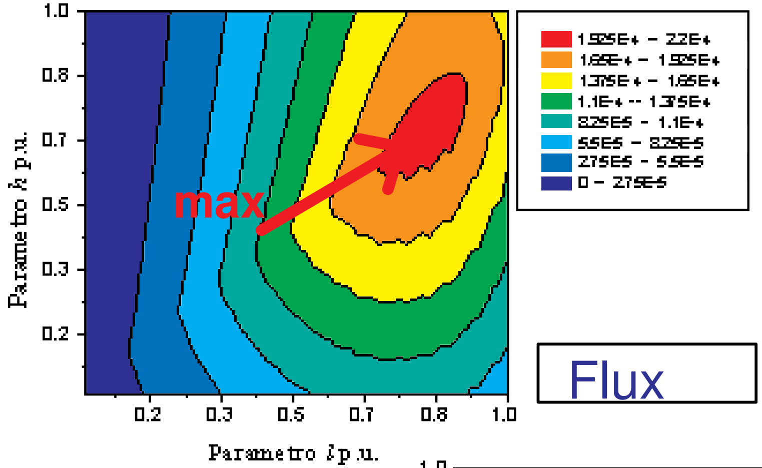


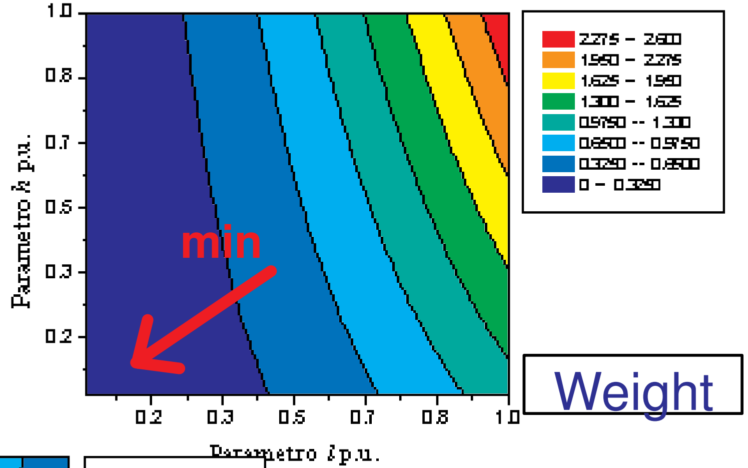


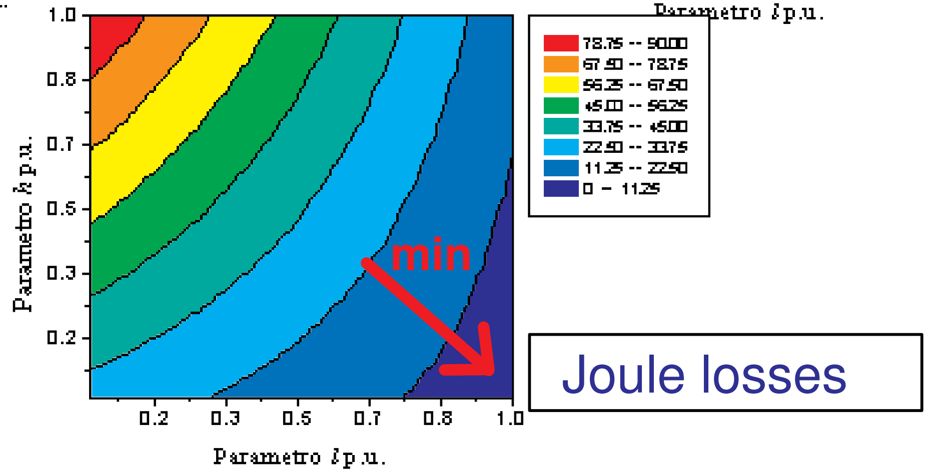
analysis can be carried out by means of magnetic circuit method taking into account nonlinearities

VOP example (2/2)

Maps of objectives

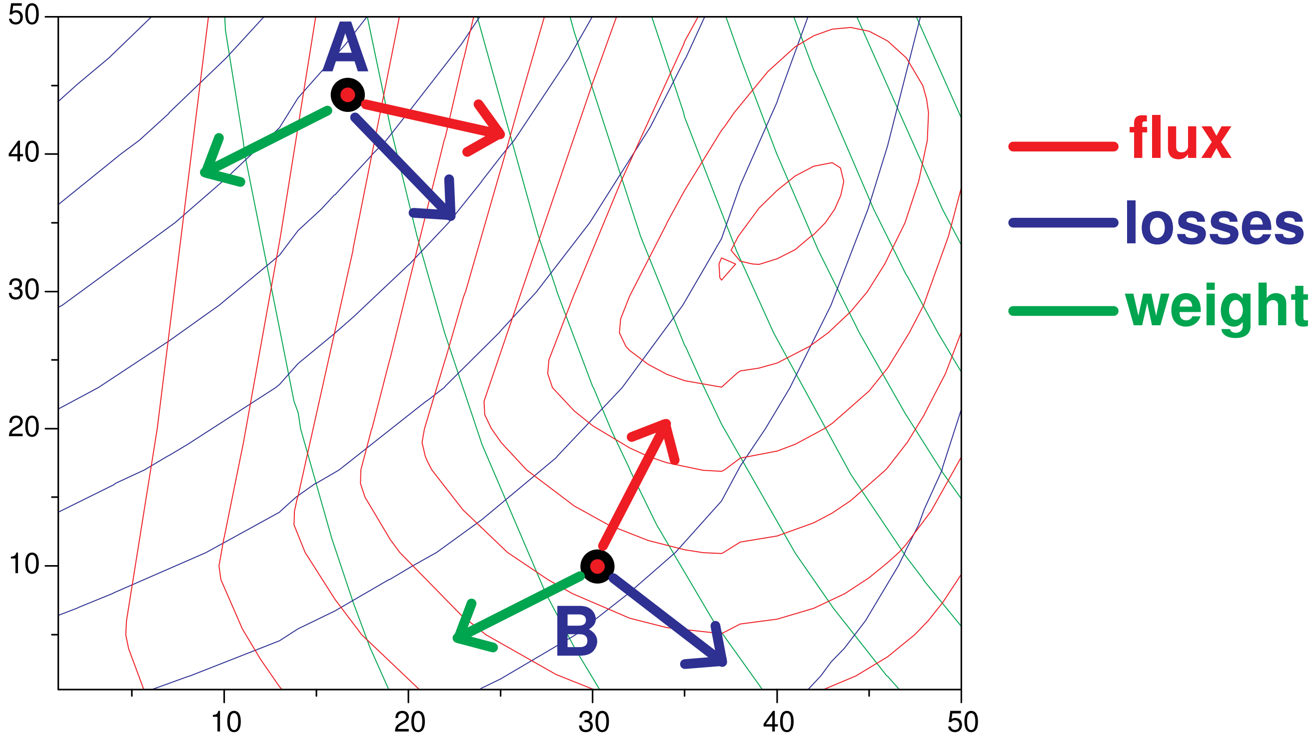




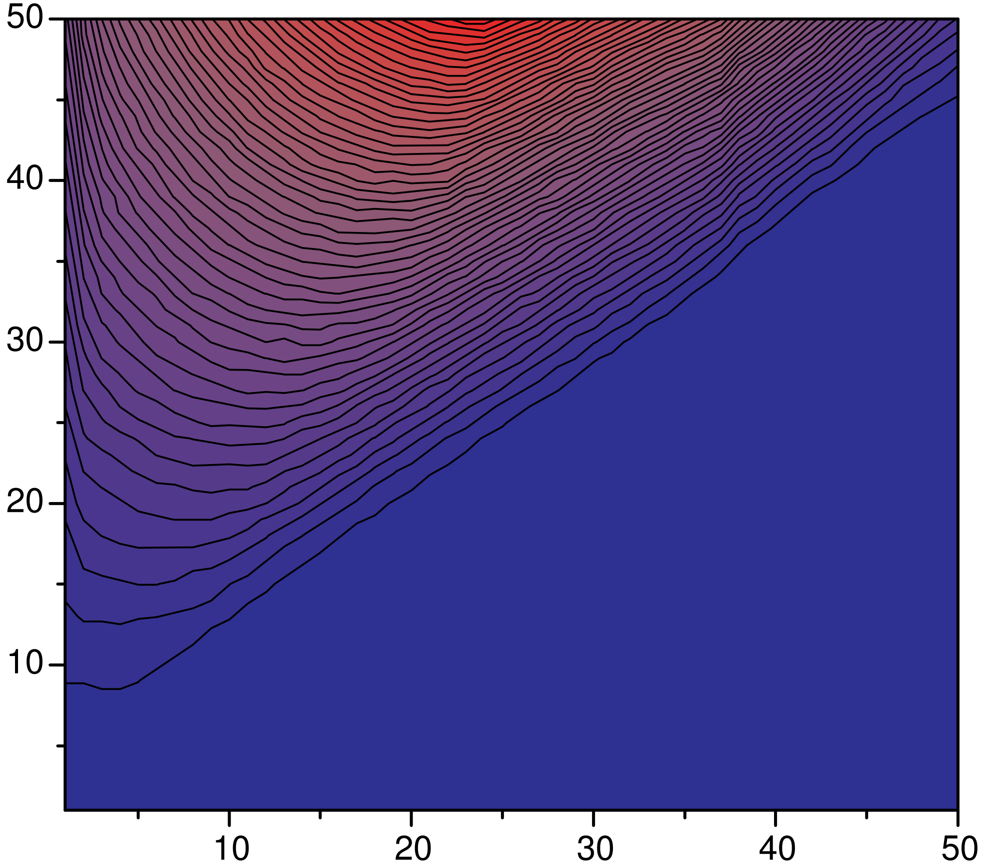


Pareto ranking (1/2)

By using a Pareto ranking scheme on the problem an identification of the Pareto front can be obtained

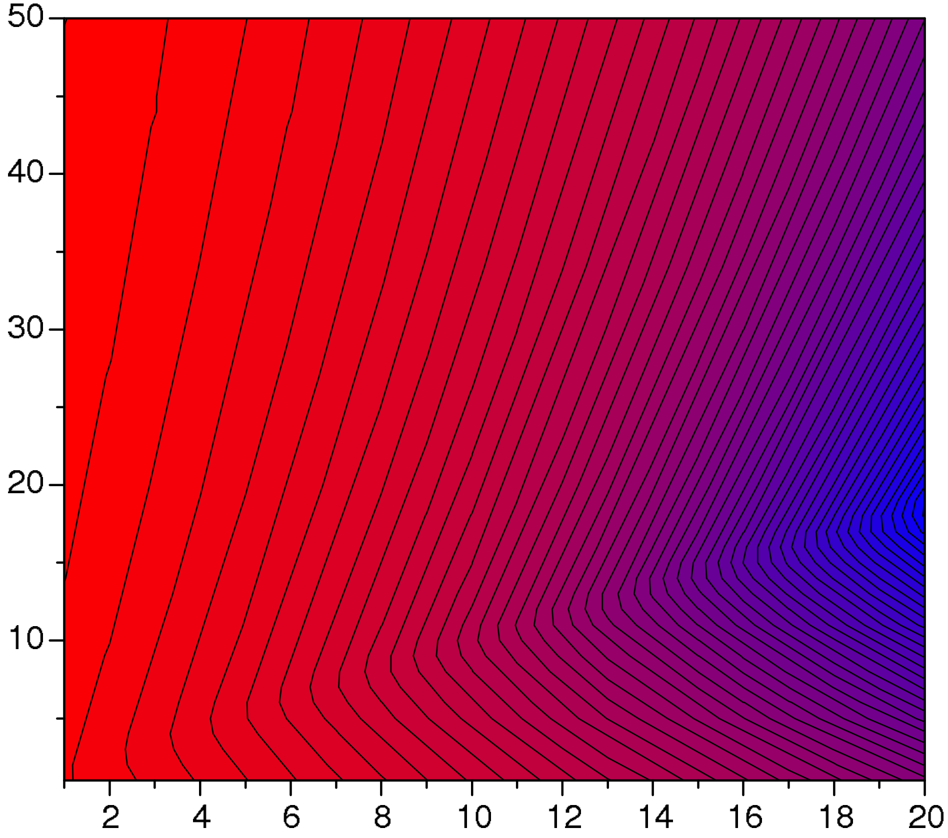


Pareto ranking (2/2)



Fuzzy aggregation

Using the fuzzy aggregation rule and setting 3 membership functions on the objectives, the map of the resulting scalar function becomes



Conclusions

- The solution of vector optimization problems is an art of compromise and tradeoff among different requirements
- While scalar optimization problems can find univocally a solution, vector ones require a criterion to pick up a "best" solution among a set of Pareto optimal ones
- Despite these difficulties, which are proper of the VOPs, research has found methods which can take advantage of the results got in scalar optimization
- In fact, both with Pareto based approaches or with aggregation ones, it is possible to use the power of evolutionary algorithms