Efficient Inclusion of Layered Media Green's Functions in Full-wave Analysis of Microstrips

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Outline

1) Aim of the work
2) Full-Wave TL Model, Enhanced Transmission Line
3) Green's Functions Decomposition
4) Computational Cost and Truncation Criteria
5) Example: Full-Wave Analysis of Microstrips
6) Future Works

Aim of the work

Integral electromagnetic formulations

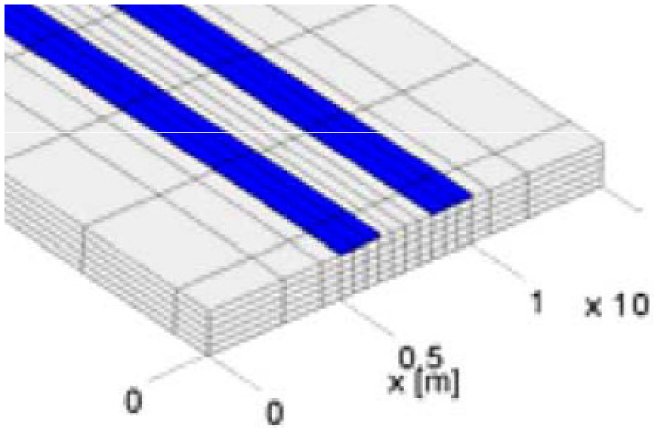
Advantage:
1) Discretization of only the conductor volumes or sometimes the conductor surfaces
2) Regularity conditions at infinity, rigorously imposed

Drawback:
Non-trivial inclusion of inhomogeneous dielectrics
Two way:
1) Discretize the dielectric domain
2) Compute the Green's Function for the domain

Aim of the work

Integral formulations: Inclusion of inhomogeneous dielectrics

1) Discretize the dielectric domain



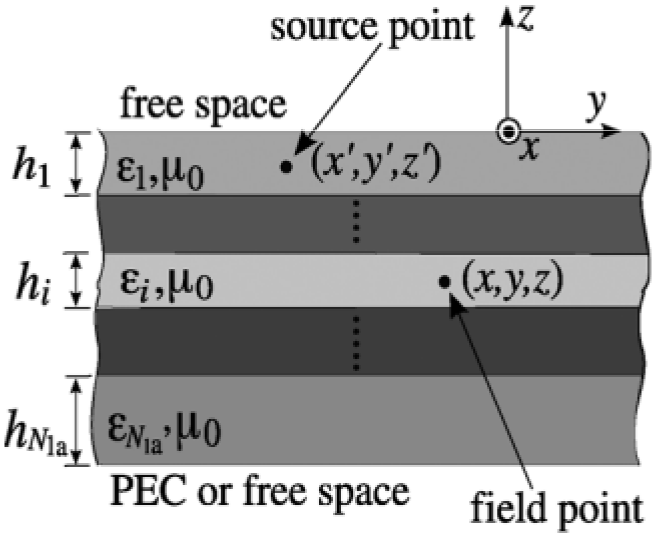
- **Any type of substrate can be included**
- **Free space Green's function, very simple**
- **High computation cost**

**Ref. G.Rubinacci, A.Tanmburrino,
IEEE Trans Antenna and Prop., 2006**

Aim of the work

Integral formulations: Inclusion of inhomogeneous dielectrics

2) Compute the Green's Function for the domain



- **Can be derived only for stratified media**
- **Can't be computed analitically**
- **Can have an high computational cost (lots of terms, Hankel functions)**

Ref. V. N. Kourkoulos and A. C. Cangellaris, IEEE Trans. Antennas Propag., 2006

Full-Wave EFIE Model

Electrical Field Integral Equation

1) **The electric field E is expressed in terms of scalar and vector potentials**

$$\boldsymbol{E} = -j\omega \boldsymbol{A} - \nabla \varphi$$

2) **The potentials are related to the "sources"**

$$\boldsymbol{A}(\boldsymbol{r}) = \mu_0 \iiint_V \underline{\underline{G}}_A(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}') \mathrm{d}V$$

$$\varphi(\boldsymbol{r}) = \frac{1}{\varepsilon_0} \iiint_V G_\phi(\boldsymbol{r},\boldsymbol{r}') \cdot \rho(\boldsymbol{r}') \mathrm{d}V$$

Ref. J. S. Zhao and W. C. Chew, IEEE Trans. on Antennas and Propagation, 2000

Full-Wave TL Model: enhanced TL model

$$\frac{\mathrm{d}\boldsymbol{I}(x)}{\mathrm{d}x} = -i\omega\boldsymbol{Q}(x), \quad \frac{\mathrm{d}\boldsymbol{V}(x)}{\mathrm{d}x} = -i\omega\boldsymbol{\Phi}(x),$$

$$\boldsymbol{\Phi}(x) = \mu_0 \int_0^l H_I(x-x')\boldsymbol{I}(x')\mathrm{d}x'$$

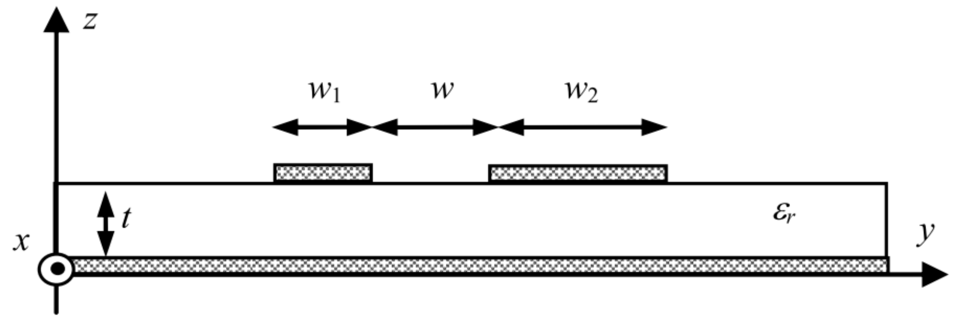
$$\boldsymbol{V}(x) = \frac{1}{\varepsilon_0}\int_0^l H_Q(x-x')\boldsymbol{Q}(x')\mathrm{d}x'.$$

$$H_I^{ik} = \frac{1}{c_i}\oint_{\Gamma_i}\mathrm{d}s_i \oint_{\Gamma_k} G_A^{xx}(s_i, s'_k; \zeta) F_i(s'_k)\mathrm{d}s'_k$$

$$H_Q^{ik} = \frac{1}{c_i}\oint_{\Gamma_i}\mathrm{d}s_i \oint_{\Gamma_k} G_\varphi(s_i, s'_k; \zeta) F_i(s'_k)\mathrm{d}s'_k$$

Valid for:

- geometrically long line
- electrically short conductor thickness
- uniform cross sections



Ref. A. G. Chiariello, A. Maffucci, G. Miano, F. Villone and W. Zamboni, IEEE Trans. on Advanced Packaging, 2008

Green's Functions for layered media

Ref. A. G. Chiariello, A. Maffucci, Journal Of Electromagnetic Analysis And Applications, 2012

$$\underline{\underline{G_A}}(r,z,z') = \frac{1}{4\pi}\int_{-\infty}^{+\infty} \underline{\underline{\tilde{G}_A}}(k_\rho) H_0^{(2)}(k_\rho r) k_\rho \mathrm{d}k_\rho$$

$$G_\varphi(r,z,z') = \frac{1}{4\pi}\int_{-\infty}^{+\infty} \tilde{G}_\varphi(k_\rho) H_0^{(2)}(k_\rho r) k_\rho \mathrm{d}k_\rho$$

Hypothesis:
Dielectric substrate infinite in the x-y plane and layered in the z direction

There are closed form expressions of the GFs $\underline{\underline{\tilde{G}_A}}$ and $\tilde{G}_\varphi$ in the spectral domain for layered media.

The numerical computation of Sommerfeld integrals is time consuming.

Green's Functions for layered media

$$\tilde{G}\left(k_\rho\right) = \tilde{G}_{as}\left(k_\rho\right) + \tilde{G}_r\left(k_\rho\right)$$

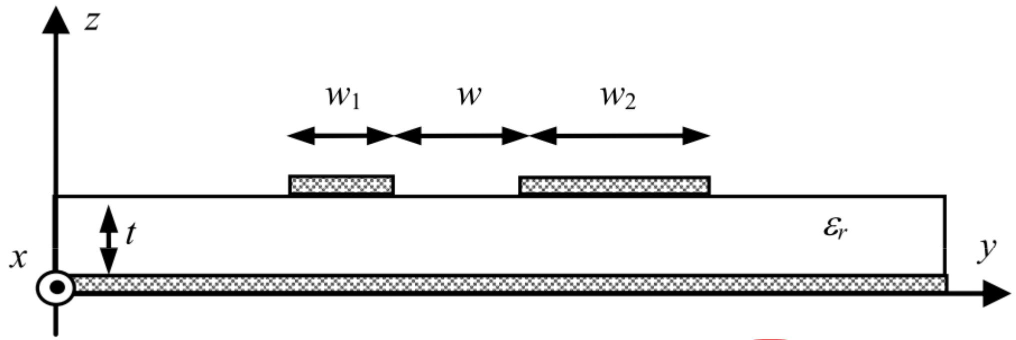
The quasi-dynamic term $\tilde{G}_{AS}(k_\rho)$ **is the asymptotic value when** $k_\rho \rightarrow \infty$

The corresponding spatial domain term dominates the near field interaction, i.e. the low frequency and low distance ranges.

The quasi-dynamic term has a simple analytical expression.

Green's Function: quasi-dynamic term

Single layer Microstrip



Static image

$$G_{Aas}^{xx}(\rho,z,z')=\frac{1}{4\pi}\frac{e^{-ikr}}{r}-\frac{1}{4\pi}\frac{e^{-ikr_1}}{r_1},$$

$$G_{\varphi as}(\rho,z,z')=\frac{1+K}{4\pi}\frac{e^{-ikr}}{r}+\frac{K^2-1}{4\pi}\sum_{n=1}^{\infty}K^{n-1}\frac{e^{-ikr_n}}{r_n}$$

Where:

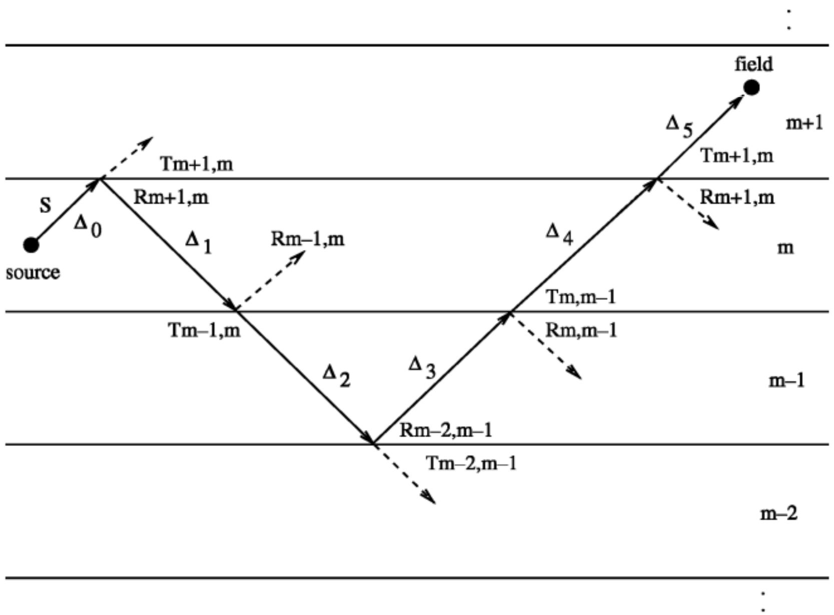
$$r=\sqrt{\rho^2+(z-z')^2}$$

$$r_n=\sqrt{\rho^2+(z+z'+2nt)^2}.$$

$$K=\frac{1-\varepsilon_r}{1+\varepsilon_r}$$

Green's Function: quasi-dynamic term

General case of multilayer media
On-chip interconnect



In the general case, the static image can be extracted using a wave tracing algorithm

Ref. F.Ling, V.Okhmatovski, B. Song, and A.Dengi, IEEE LAWP, 2007

Green's Functions for layered media

$$\tilde{G}(k_\rho) = \tilde{G}_{as}(k_\rho) + \tilde{G}_r(k_\rho)$$

The dynamic term $\tilde{G}_r(k_\rho)$ is approximated in the spectral domain in terms of functions that lead to closed-form expressions of Sommerfeld integral

$$\tilde{G}_r(k_\rho) = \sum_{k=1}^{N} a_k e^{-b_k k_{zr}}$$

$$\tilde{G}_r(k_\rho) = \sum_{k=1}^{N} \frac{a_k}{k_\rho^2 - p_k^2},$$

Ref[1] M. I. Aksun and G. Dural, IEEE Trans. on AP, Vol.53, No. 11, 2005

Ref[2] V. I. Okhmatovski and A. C. Cangellaris,IEEE Trans. on AP, Vol.54, No. 7, 2002

Ref[3] V.N. Kourkoulos and A.C. Cangellaris, IEEE Trans. On AP, Vol.54, (2006).

Ref[4] F. Mesa, R.R. Boix, F. Medina, IEEE Trans. MTT, Vol.56, No.7, (2008),

Green's Functions for layered media

Spatial domain

$$G(\rho) = G_{as}(\rho) + G_r(\rho)$$

The **dynamic term** $G_r(\rho)$ in the spatial domain is approximated by a combination of spherical waves and cylindrical waves

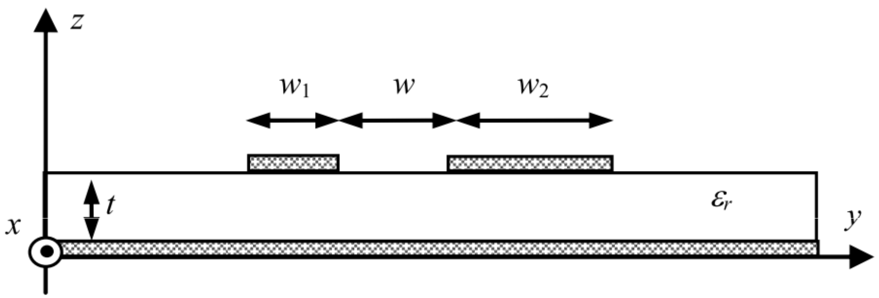
$$G_r(\rho) = \sum_{k=1}^{N} a_k \frac{e^{-jkr_{bk}}}{r_{bk}} \quad ;$$

$$G_r(\rho) = -\frac{i}{4} \sum_{k=1}^{N} a_k H_0^{(2)}(p_k r)$$

The computation of the dynamic term can be time consuming

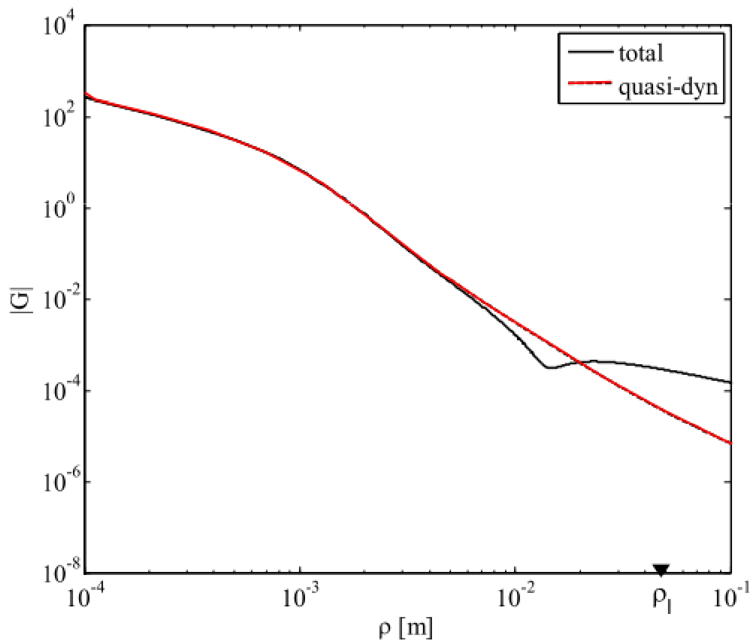
Green's Function: quasi-dynamic term

Single layer Microstrip

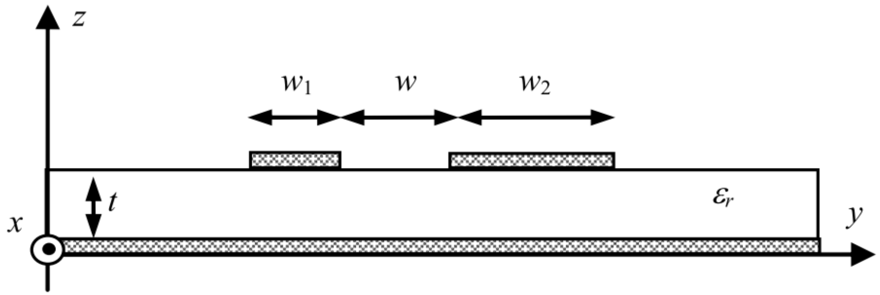


$$G_{\varphi as}(\rho,z,z') = \frac{1+K}{4\pi}\frac{e^{-ikr}}{r} + \frac{K^2-1}{4\pi}\sum_{n=1}^{\infty} K^{n-1}\frac{e^{-ikr_n}}{r_n}$$

When the quasi-dynamic term is a good approximation for the total Green function in the full-wave simulation?



Green's Function: approximation criteria



$$k_0 t\sqrt{\varepsilon_r - 1} < 0.7$$

The first surface wave TE_1 is below cutoff

$$l < 2 / k_0\sqrt{\varepsilon_r}$$

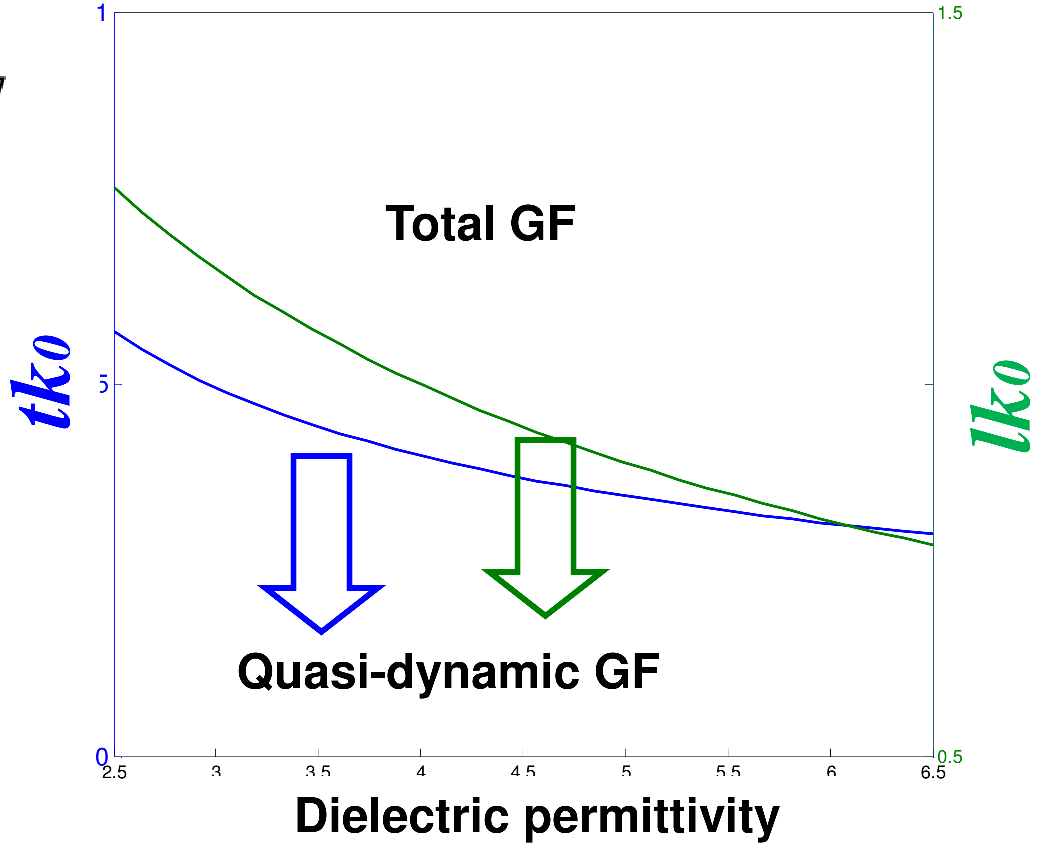
The quasi-dynamic term is dominant over the Fabry-Perot leaky wave

Green's Function: approximation criteria map

$$k_0 t \sqrt{\varepsilon_r - 1} < 0.7$$

$$k_0 t < \frac{0.7}{\sqrt{\varepsilon_r - 1}}$$

dielectric thickness

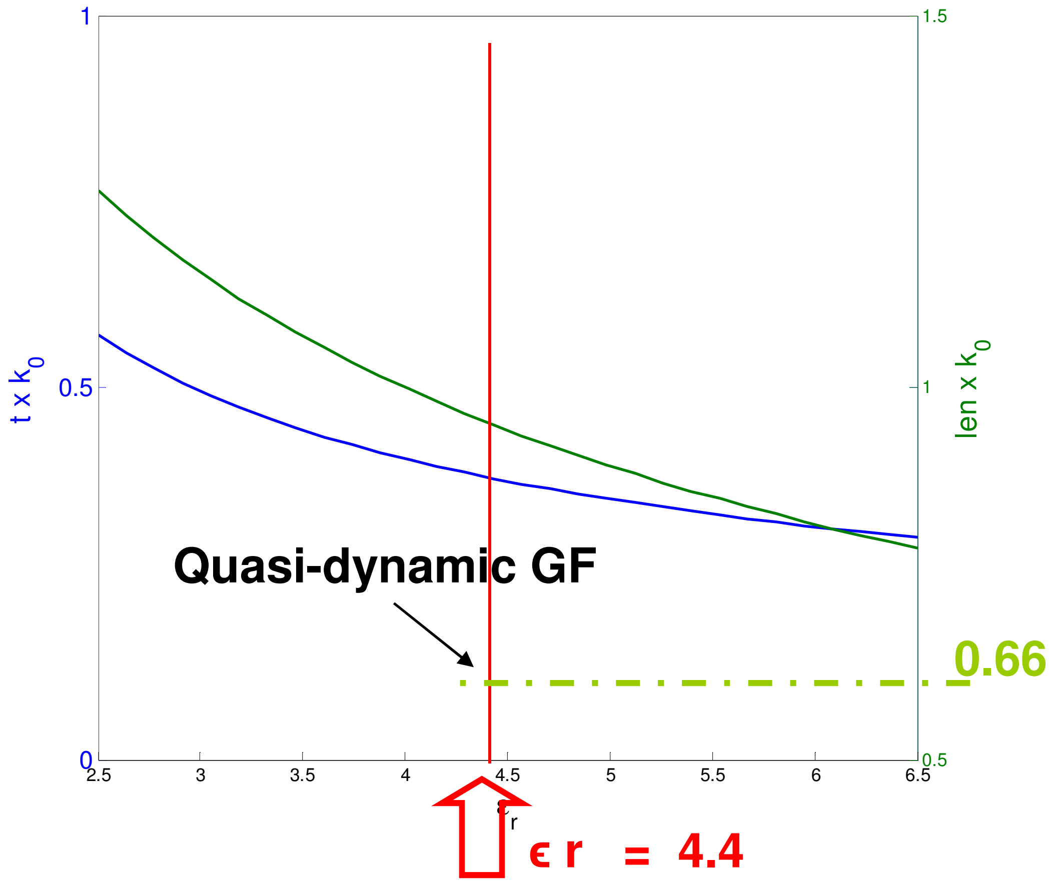


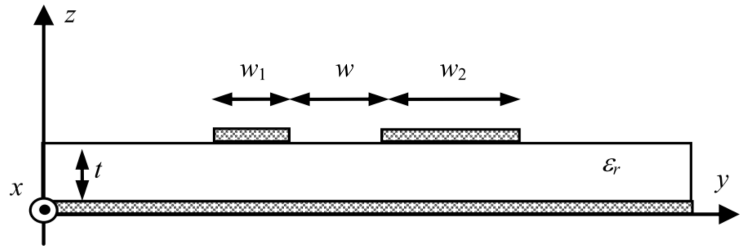
$$l < 2 / k_0 \sqrt{\varepsilon_r}$$

$$k_0 l < \frac{2}{\sqrt{\varepsilon_r}}$$

line length

Green's Function: criteria for quasi-dyn term satisfied





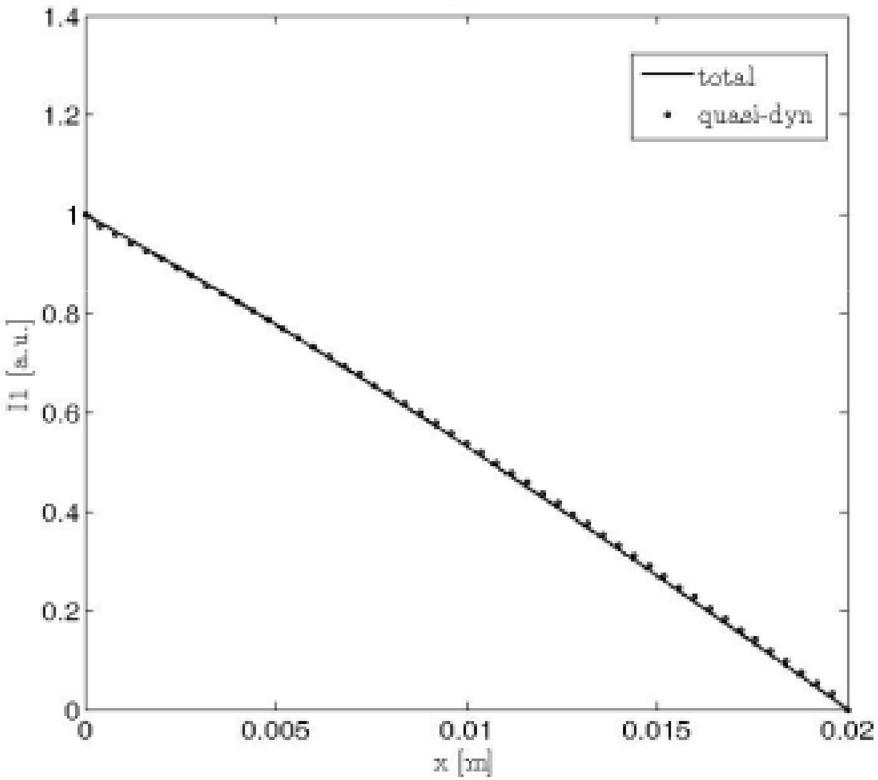
w 1 = 0.25 mm
w 2 = 0.50 mm
s = 0.228 mm
ϵ r = 4.4

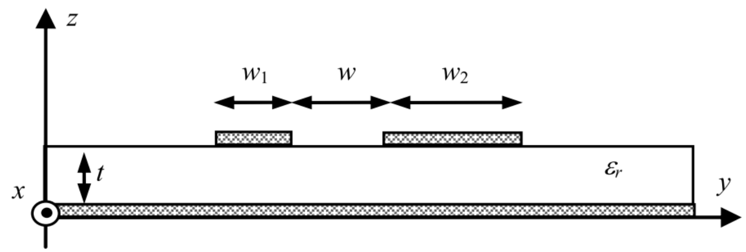
f=1GHz → k0 =ω/c = 3.33

t=0.60 mm → **t*k0 = 3e-3** →**ok**

L= 0.20 m → **L*k0 = 0.66** → **ok**

Current distribution: quasi-dyn GF vs total





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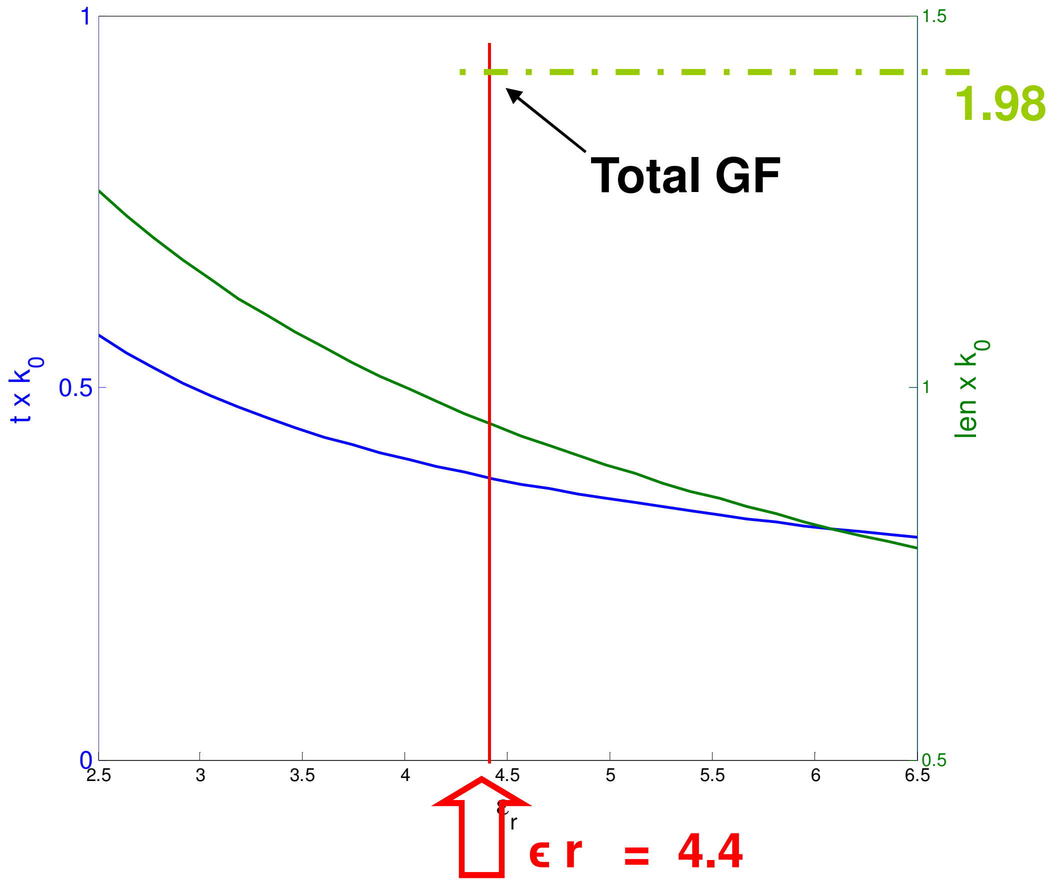
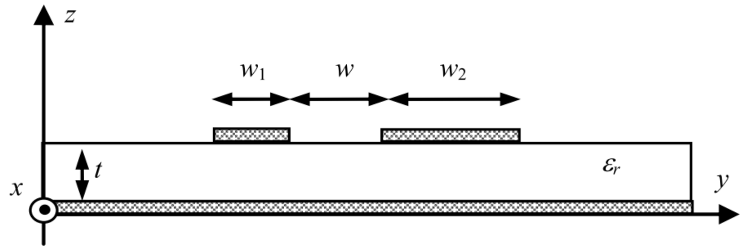
Green’s Function: criteria for quasi-dyn term NOT satisfied





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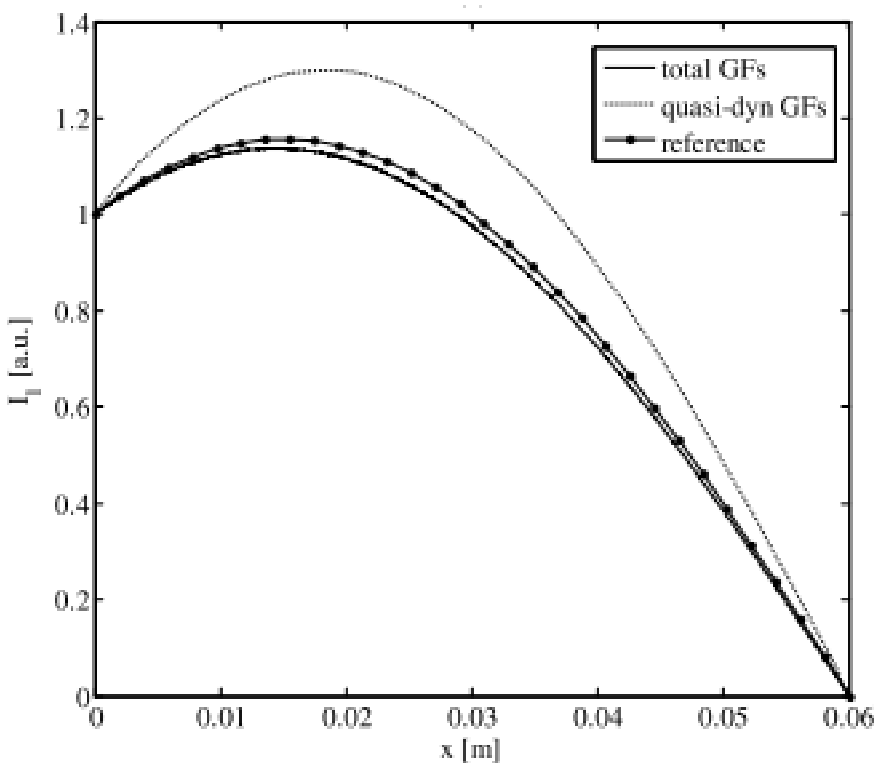
s = 0.228 mm
ϵ r = 4.4

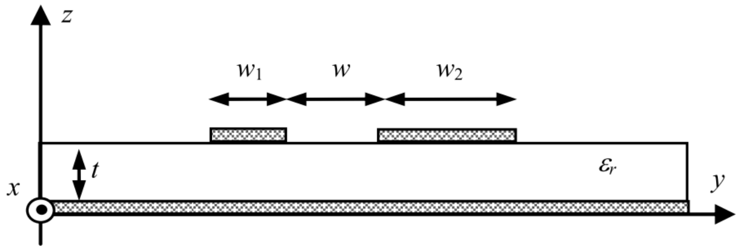
f=1GHz → k0 =ω/c = 3.33

t=0.60 mm → **t*k0 = 3e-3** → **ok**

L= 60 m → **L*k0 = 1.98** → **no**

Current distribution: quasi-dyn GF vs total





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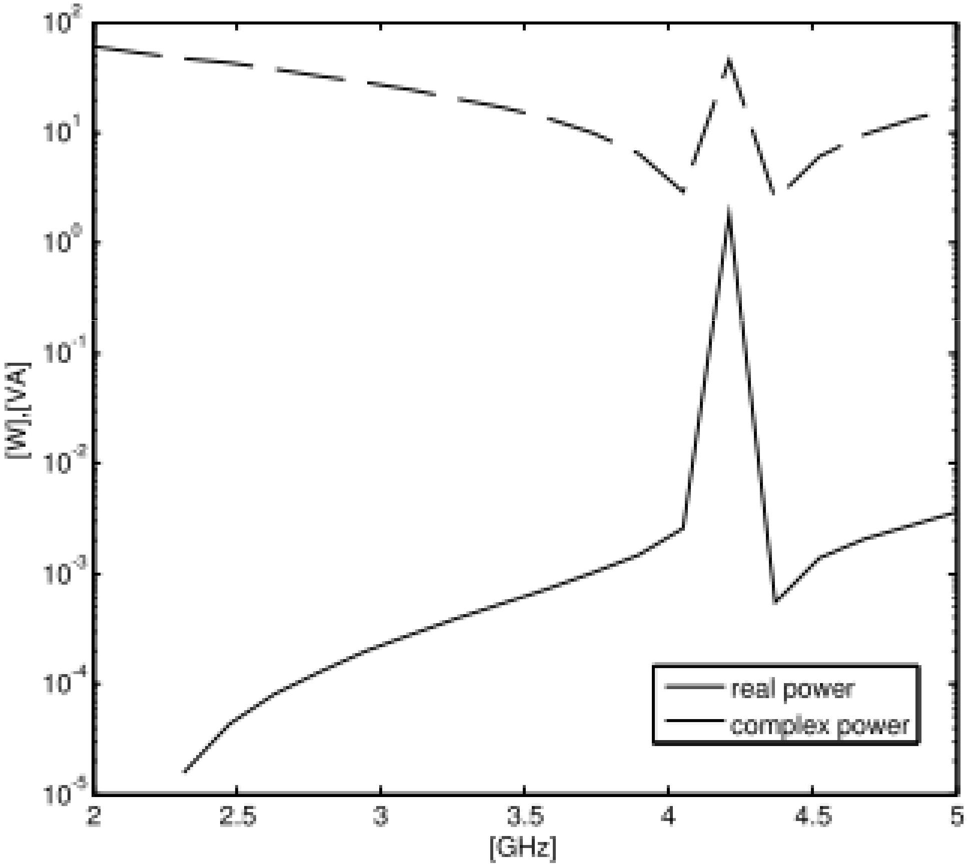
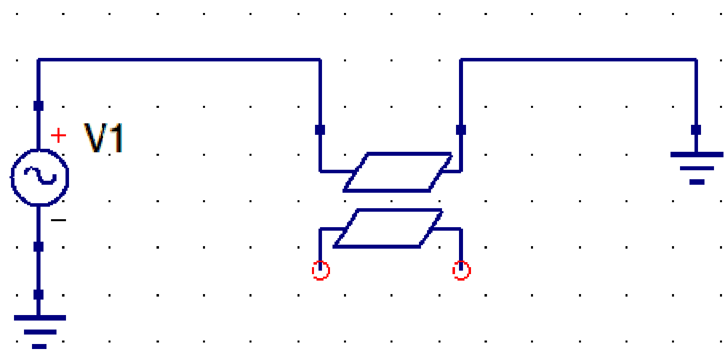
f=1GHz → k0 =ω/c = 3.33

t=0.60 mm → **t*k0 = 3e-3** →**ok**

L= 60 m → **L*k0 = 1.98** → **no**

Absorbed real power

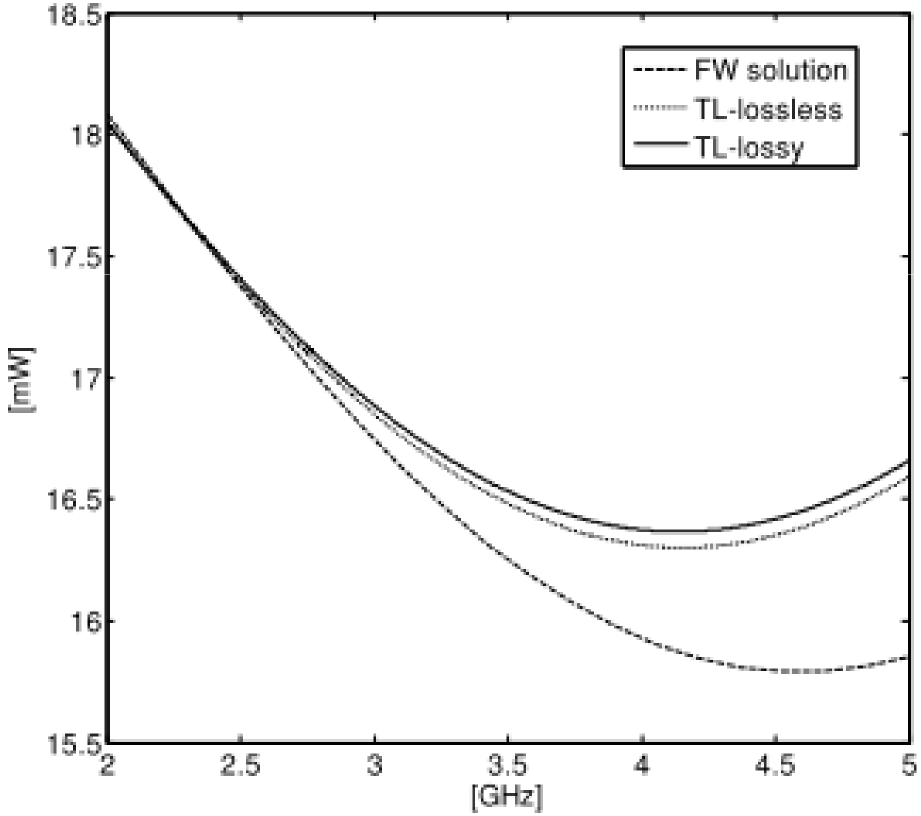


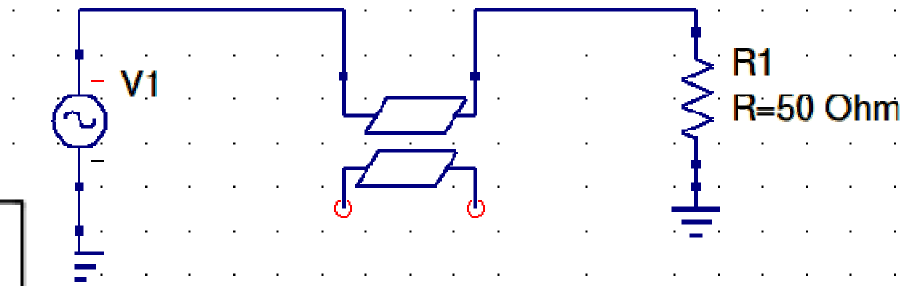


w 1 = w 2 = 0.33 mm
s = 2*w1
ϵ r = 4.2
f = 1GHz
t = 0.20 mm
L= 10 mm

Full-wave effect: excitation of high parasitic modes

Absorbed real power: including loss





w 1 = w 2 = 0.33 mm
s = 2*w1
ϵ r = 4.2
f = 1GHz
t = 0.20 mm
L= 10 mm

The FW effect are much more important than the dielectric and conductor losses.

Conclusions

1. **The quasi-dynamic approximation lowers the computational cost of the Green Functions**
2. **Two simple criteria are derived to establish whether the quasi-dynamic terms approximate the complete Green's functions for full-wave analysis of microstrip. The criteria involve the dielectric thickness and the line length.**
3. **When the criteria are satisfied, the quasi-dynamic terms take into account correctly the Full Wave effects**

Future Works

1. **Extend the criteria to the multi layers case (on-chip interconnect)**
2. **Analyze non planar structure, Vias etc.**