Improving the Accuracy of Rational Macromodels via Mode-Revealing Transformation

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Background

- Passive macromodeling of linear devices and systems is based on rational function approximation.

- The current practice for admittance-based modeling leads to models that are
  - Accurate in applications with low-impedance terminations.
  - Sometimes very inaccurate with high-impedance terminations.

- Problem:
  - Error magnification
  - Occurs for **Y** with large condition number (case-dependent).

Existing solutions

- Modal Vector Fitting (model extraction)
  - Complex implementation
  - CPU intensive
  - Patented

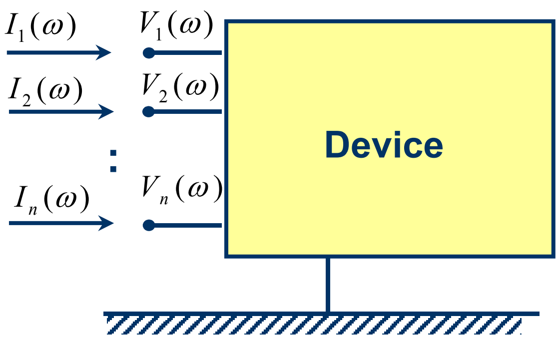
- Modal Perturbation (passivity enforcement)
  - Complex implementation
  - CPU intensive
  - Patented

This presentation: Alternative approach

Admittance-based modeling

- Admittance matrix **Y** defines the current response **i**(ω) due to a voltage application **v**(ω).

$$\mathbf{i}(\omega) = \mathbf{Y}(\omega)\mathbf{v}(\omega)$$



- Pole-residue model

$$\mathbf{Y}(\omega) \approx \mathbf{R}_0 + \sum_{p=1}^{P} \frac{\mathbf{R}_p}{j\omega - a_p}$$

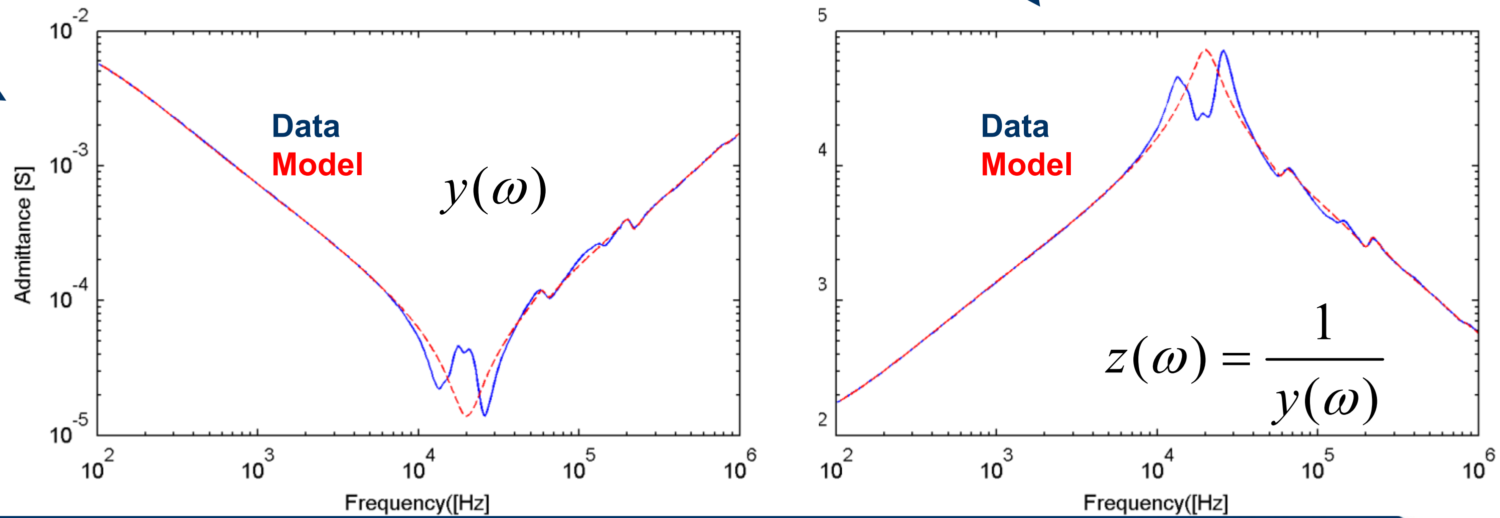
- The model is most accurate when applied in simulations with voltage applications, i.e. low-impedance terminations.

Error magnification: single terminal case

$$i(\omega) = y(\omega)v(\omega) \qquad y(\omega) \approx r_0 + \sum_{p=1}^{P} \frac{r_p}{j\omega - a_p}$$

- Direct fitting of y(ω) gives a model minimizing global error.
  - Relative error is large where y(ω) is small in magnitude.
  - Large error magnifications result when applying current to the model since the voltage response is given by z=y^{-1}(ω) which is large where y(ω) is small,

***P*=10**



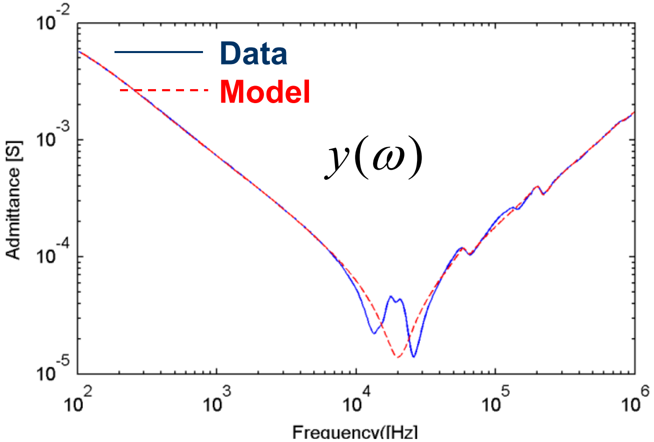
Avoiding error magnification:

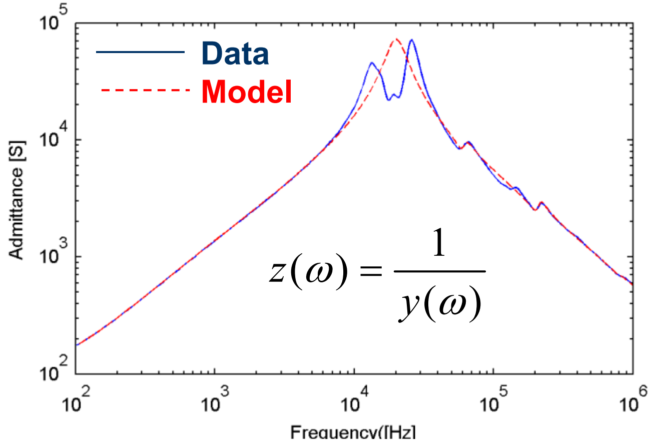
- Introduce least squares weighting equal to the inverse of the $|y(\omega)|$ → <u>relative error control</u>

$$\text{weight}(\omega) = \frac{1}{|y(\omega)|}$$

$$\text{weight}(\omega) = 1$$

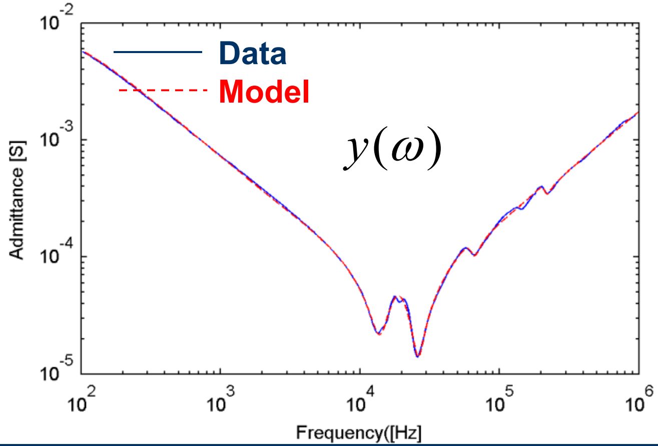
(no weighting)

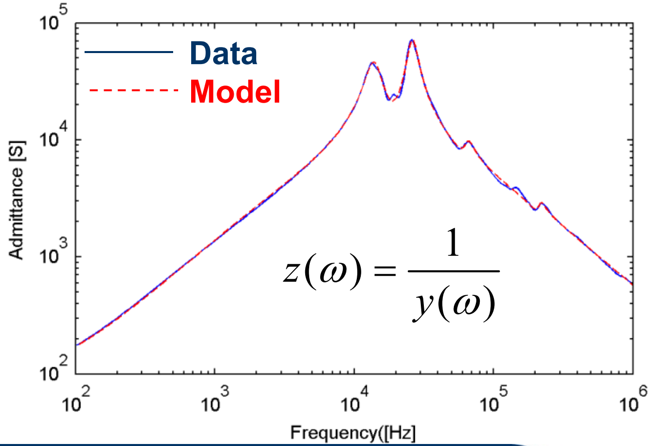




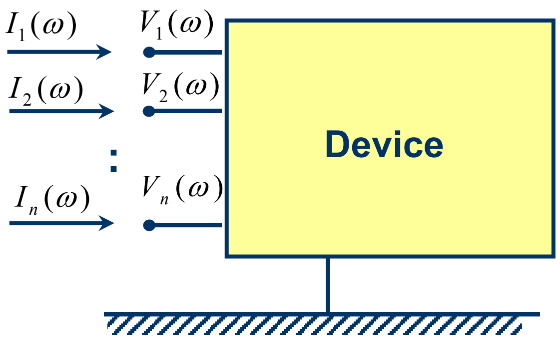
P=10

$$\text{weight}(\omega) = \frac{1}{|y(\omega)|}$$





Error magnification: Multi-terminal case



$$\mathbf{i}(\omega) = \mathbf{Y}(\omega)\mathbf{v}(\omega)$$

$$\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{16} \\ y_{21} & y_{12} & & \\ \vdots & & \ddots & \\ y_{61} & & & y_{66} \end{bmatrix}$$

$$\mathbf{Y}(\omega) \approx \mathbf{R}_0 + \sum_{p=1}^{P} \frac{\mathbf{R}_p}{j\omega - a_p}$$

- How to avoid error magnifications?
  - "Intuitive" approach:
    Apply inverse weighting to individual elements

$$\text{weight}_{ij}(\omega) = \frac{1}{|\mathbf{Y}_{ij}(\omega)|}$$

  - This <u>not</u> the answer
    Reason: The matrix inverse is not equal to the matrix of inverse elements, $(\mathbf{Y}^{-1})_{ij} \neq (\mathbf{Y}_{ij})^{-1}$

Error magnification: The modal view

- The matrix inverse implies inverse eigenvalues

Eigenvector matrix

Eigenvalue matrix (diagonal)

$$\mathbf{Y}(\omega) = \mathbf{T}_Y(\omega)\mathbf{\Lambda}_Y(\omega)\mathbf{T}_Y^{-1}(\omega)$$

$$\begin{aligned}
&\mathbf{Y}^{-1}(\omega) \\
&= \left(\mathbf{T}_Y(\omega)\mathbf{\Lambda}_Y(\omega)\mathbf{T}_Y^{-1}(\omega)\right)^{-1} \\
&= \mathbf{T}_Y(\omega)\mathbf{\Lambda}_Y^{-1}(\omega)\mathbf{T}_Y^{-1}(\omega)
\end{aligned}$$

The eigenvalues of **Y** are equal to the inverse eigenvalues of $\mathbf{Y}^{-1}$

The fitting should be performed in such way that the accuracy of the eigenvalues is preserved in the *relative* sense.

Mode Revealing Transformation

- Apply a suitable **orthogonal similarity transformation** to **Y**

$$\tilde{\mathbf{Y}} = \mathbf{T}^{-1}\mathbf{Y}\mathbf{T}$$

- $\tilde{\mathbf{Y}}$ and **Y** have identical eigenvalues (similarity transformation) $\lambda_{\tilde{\mathbf{Y}}} = \lambda_{\mathbf{Y}}$
- Objective: Make the eigenvalues are more visible in $\tilde{\mathbf{Y}}$ than in **Y**.
- Fitting the elements of $\tilde{\mathbf{Y}}$ using inverse weighting for its elements gives high accuracy for all eigenvalues of **Y**.
- $\tilde{\mathbf{Y}}$ and **Y** have identical passivity characteristics (orthogonal similarity transformation).

$$\lambda_{\mathrm{Re}\{\tilde{\mathbf{Y}}\}} = \lambda_{\mathrm{Re}\{\mathbf{Y}\}}$$

Computing a suitable $\mathbf{T}_0$

- Identify frequency ω_0 where the ratio between the largest and smallest eigenvalue in **Y**(ω) is maximum.

- Calculate eigenvalue decomposition at $\omega = \omega_0$

$$\mathbf{Y}(\omega_0) = \mathbf{T}(\omega_0)\mathbf{\Lambda}(\omega_0)\mathbf{T}^{-1}(\omega_0)$$

- Modify **T** → $\mathbf{T}_0$
  - Rotate eigenvectors to minimize imaginary part
  - Discard imaginary part
  - Enforce orthogonality

$$\mathbf{T}_0 = \mathbf{QR},\ \ \mathbf{Q} \rightarrow \mathbf{T}_0$$

- Apply $\mathbf{T}_0$ to **Y** at all frequencies as a similarity transformation,

$$\tilde{\mathbf{Y}}(\omega) = \mathbf{T}_0^{-1}\,\mathbf{Y}(\omega)\,\mathbf{T}_0$$

Model Extraction

- Fit a rational model to $\tilde{\mathbf{Y}}$
  - Vector Fitting w/ relaxation and sparse utilization
  - Relative error control (inverse magnitude weighting of each $\tilde{\mathbf{Y}}_{ij}$)

$$\tilde{\mathbf{Y}}(\omega) \approx \tilde{\mathbf{R}}_0 + \sum_{p=1}^{P} \frac{\tilde{\mathbf{R}}_p}{j\omega - a_p}$$

- Enforce passivity of model
  - Residue perturbation
  - Relative error control

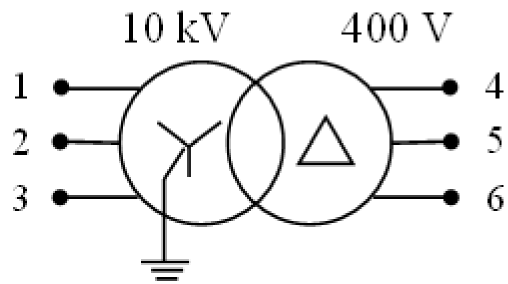
$$\tilde{\mathbf{R}}_p \to \tilde{\mathbf{R}}_p + \Delta\tilde{\mathbf{R}}_p \ , p = 0 \dots P$$

such that $\lambda_{\mathrm{Re}\{\tilde{\mathbf{Y}}\}} > 0 \ \forall \omega$

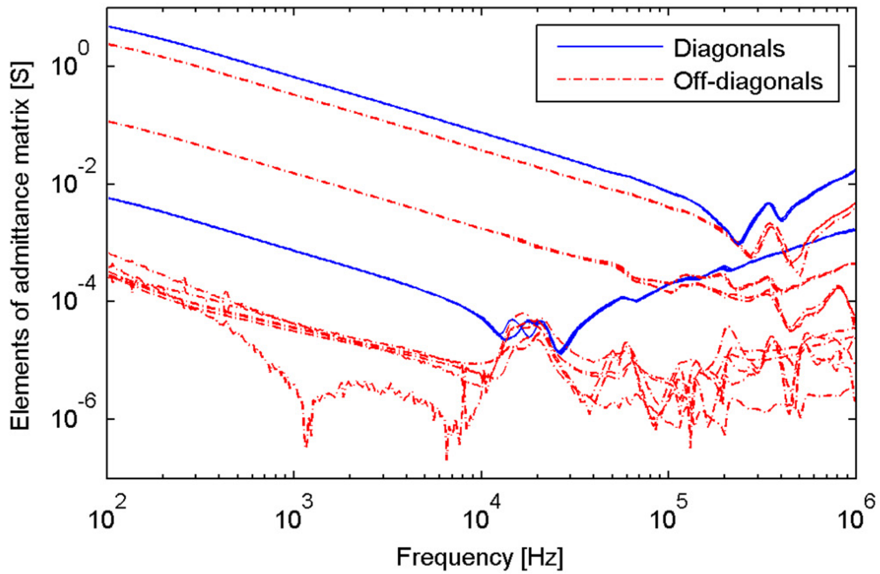
- Apply inverse similarity transform to the model

$$\mathbf{R}_p = \mathbf{T}_0 \tilde{\mathbf{R}}_p \mathbf{T}_0^{-1} \ , p = 0 \dots P$$

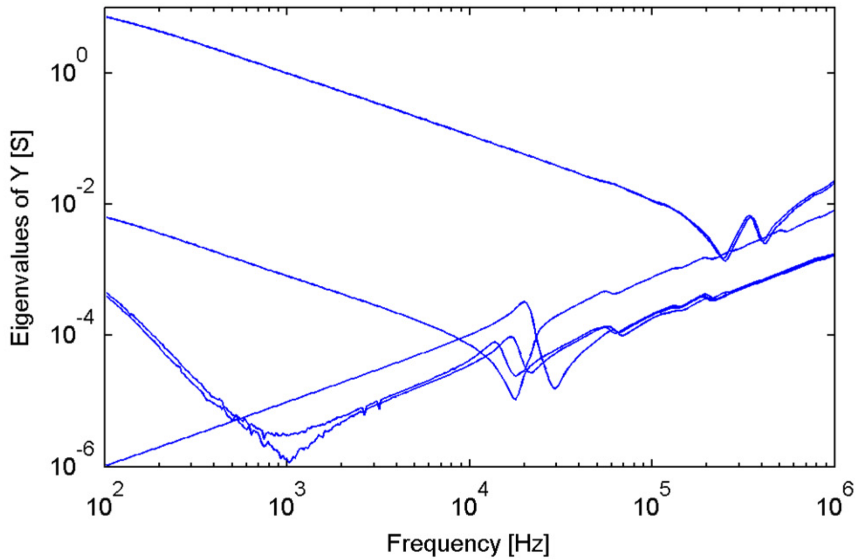
Example: 30 kVA distribution transformer



Elements of **Y**



Eigenvalues of **Y**



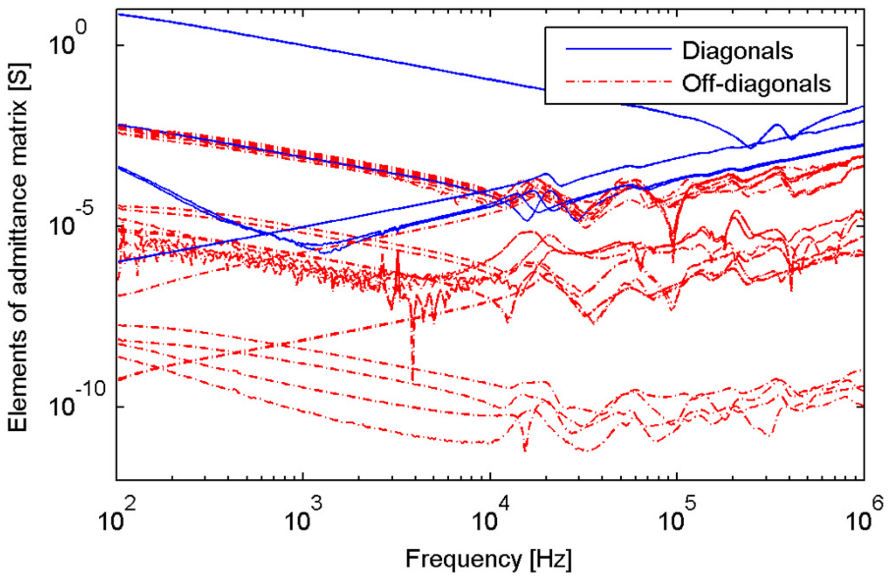
Applying Mode-Revealing Transformation

$$\omega_0 = 2\pi \cdot 100\,\mathrm{Hz}$$

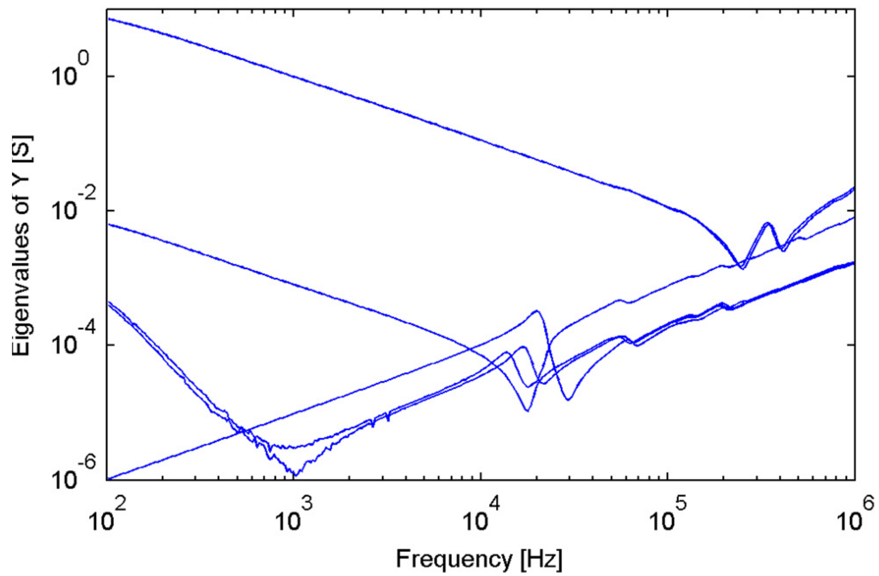
$$\tilde{\mathbf{Y}}(\omega) = \mathbf{T}_0^{-1}\,\mathbf{Y}(\omega)\,\mathbf{T}_0$$

- Compute eigenvector matrix $\mathbf{T}_0$ of $\mathbf{Y}(\omega_0)$
- Rotate $\mathbf{T}_{0,i}$,i=1..n, discard imaginary part
- Orthogonalization

Elements of $\tilde{\mathbf{Y}}$



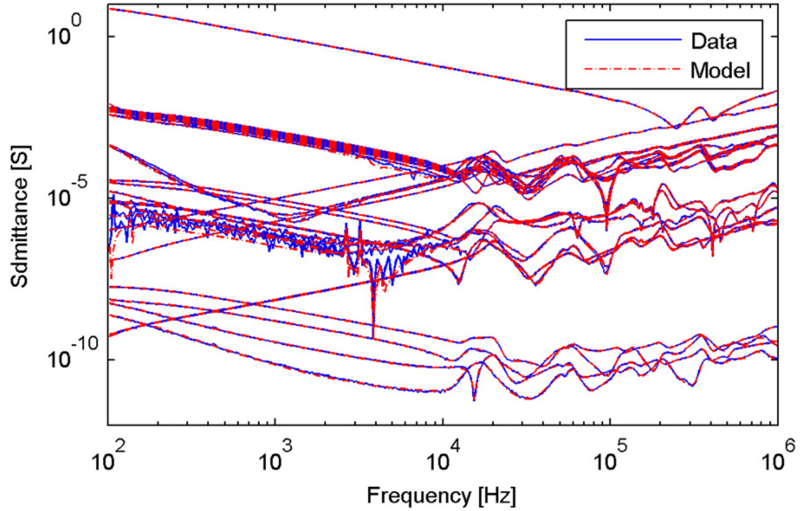
Eigenvalues of **Y**



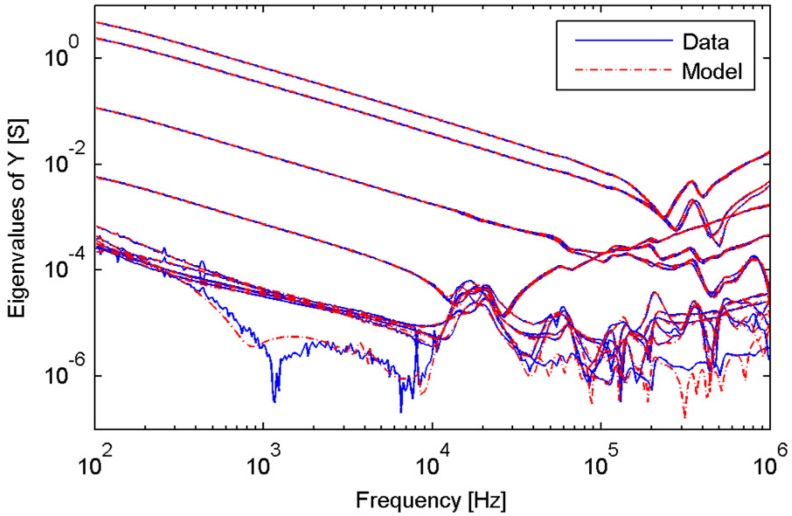
The diagonal elements of $\tilde{\mathbf{Y}}$ resemble the eigenvalues of **Y**

Rational Modeling

Elements of $\tilde{\mathbf{Y}}$



Elements of **Y**



- Fit a rational model to $\tilde{\mathbf{Y}}$

$$\tilde{\mathbf{Y}}(\omega) \approx \mathbf{R}_0 + \sum_{p=1}^{P} \frac{\mathbf{R}_p}{j\omega - a_p} \qquad (P=60)$$

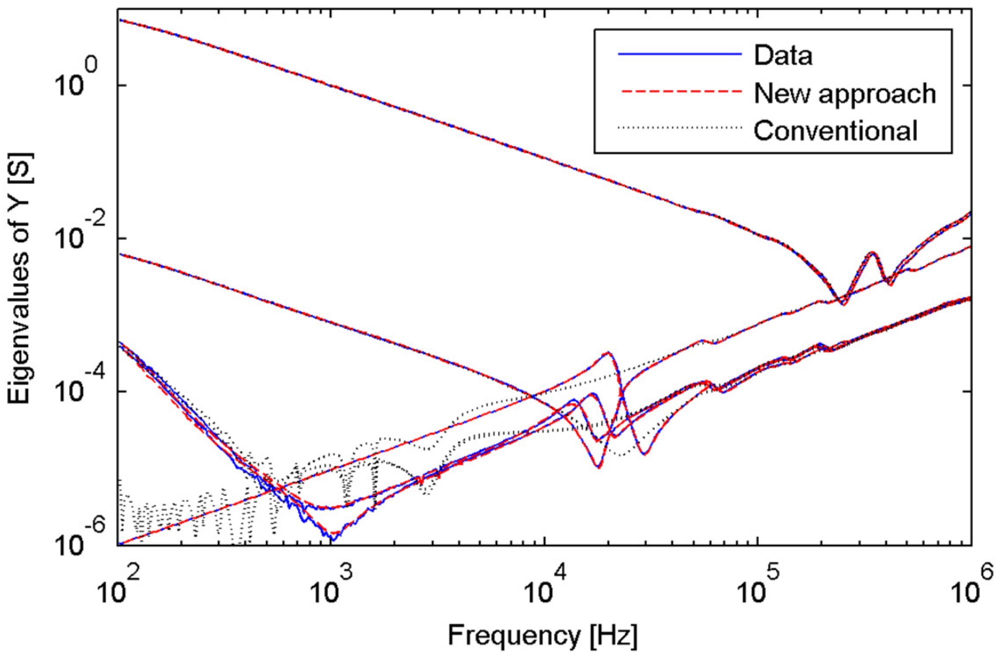
Algorithm: Vector Fitting w/inverse LS weighting

$$\mathrm{weight}_{ij}(\omega) = \frac{1}{|\tilde{\mathbf{Y}}_{ij}(\omega)|}$$

- Transform model back to the "physical domain"

$$\mathbf{R}_p = \mathbf{T}_0 \tilde{\mathbf{R}}_p \mathbf{T}_0^{-1} \quad , p = 0 \ldots P$$

Check on eigenvalues



Eigenvalues of **Y**

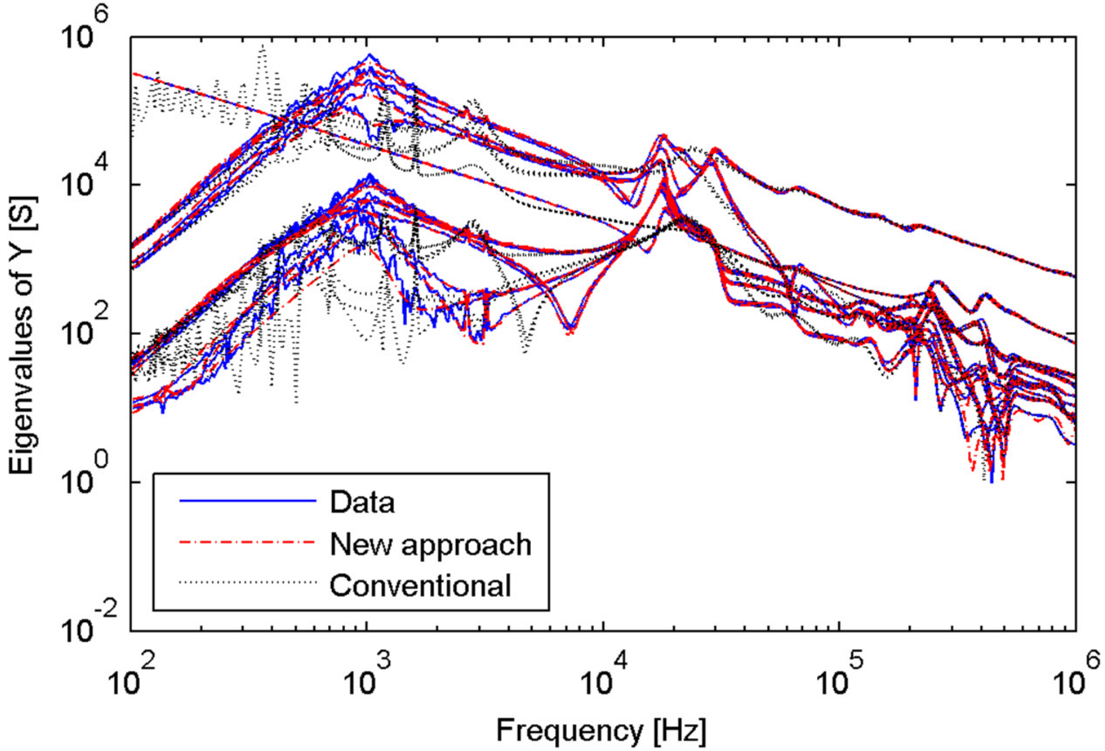
New approach

→ retains accuracy of the eigenvalues of **Y**

Conventional approach (direct fitting of **Y** w/ inverse weight)

→ corrupts the small eigenvalues

Model Accuracy With Current Application



$$\mathbf{Z}(\omega) = \mathbf{Y}^{-1}(\omega)$$

New approach

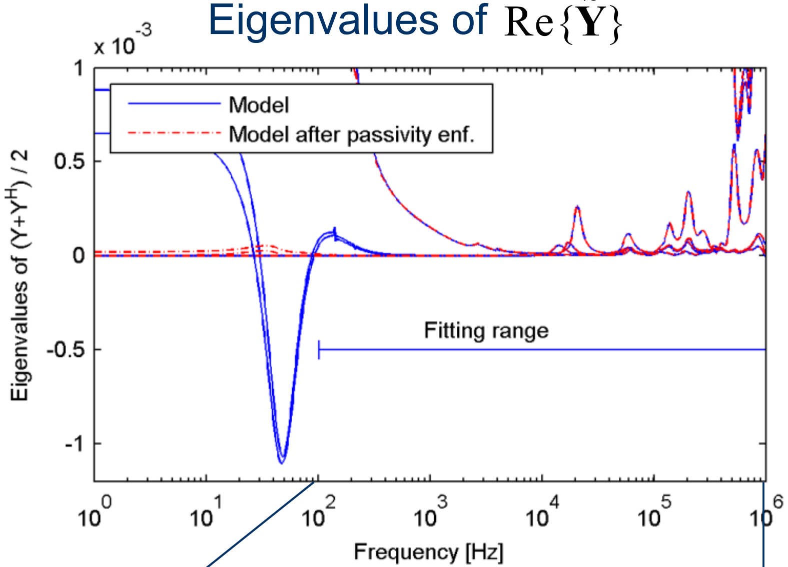
→ "Accurate" result

Conventional approach

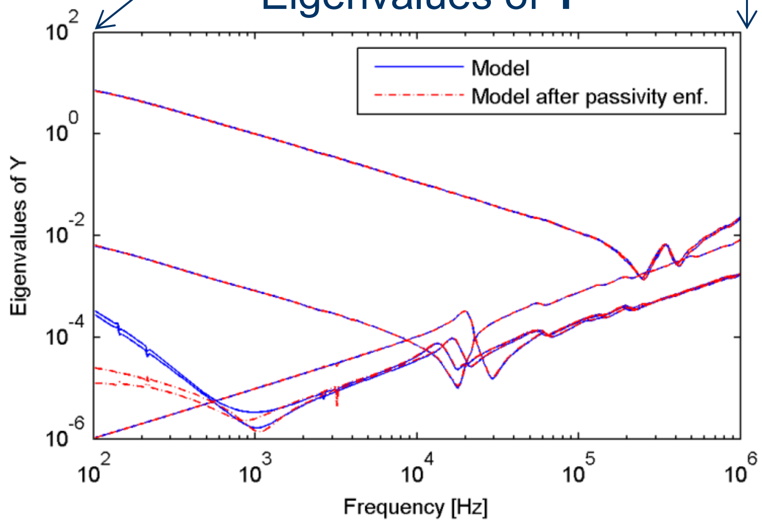
→ Large error magnifications

Passivity

Eigenvalues of $\mathrm{Re}\{\tilde{\mathbf{Y}}\}$



Eigenvalues of **Y**



- Enforce passivity of $\tilde{\mathbf{Y}}$

  Algorithm: Residue perturbation w/inverse LS weighting

$$\mathrm{weight}_{ij}(\omega) = \frac{1}{|\tilde{\mathbf{Y}}_{ij}(\omega)|}$$

- Transform model back to the "physical domain"

$$\mathbf{R}_p = \mathbf{T}_0 \tilde{\mathbf{R}}_p \mathbf{T}_0^{-1} \quad , p = 0 \ldots P$$

Conclusions

- A new method has been proposed for pole-residue modeling of the terminal admittance matrix, **Y**.
- Relies on a similarity transformation obtained from the eigenvector matrix of **Y**.
  - Preserves symmetry, eigenvalues and passivity of **Y**.
  - The eigenvalues of the transformed matrix $\tilde{\mathbf{Y}}$ are more observable than in the original **Y** (mode-revealing transformation).
  - → Model extraction and passivity enforcement of $\tilde{\mathbf{Y}}$ by standard methods preserves accuracy of eigenvalues.
- The rational model is less prone to error magnifications than with conventional approach.
- Simple to implement, fast.