Method of Moment Solution of SVS-EFIE for 2D Transmission Lines of Complex Cross-Sections

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Outline

- **Integral Equation Approaches for RL extraction**
  - Volume-IE
  - Surface-Volume-Surface IE
- **Method of Moment for SVS-EFIE**
  - Surface and volume meshing
  - Matrix structure
- **Numerical results**
  - Circular conductor: SVS-EFIE vs. Analytic Solution
  - Differential pair: SVS-EFIE vs. Volume-IE
  - Coaxial cable: SVS-EFIE vs. Volume-IE

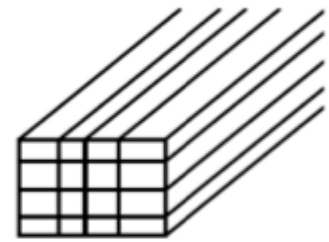
Traditional Volume-EFIE

Volume EFIE ➔

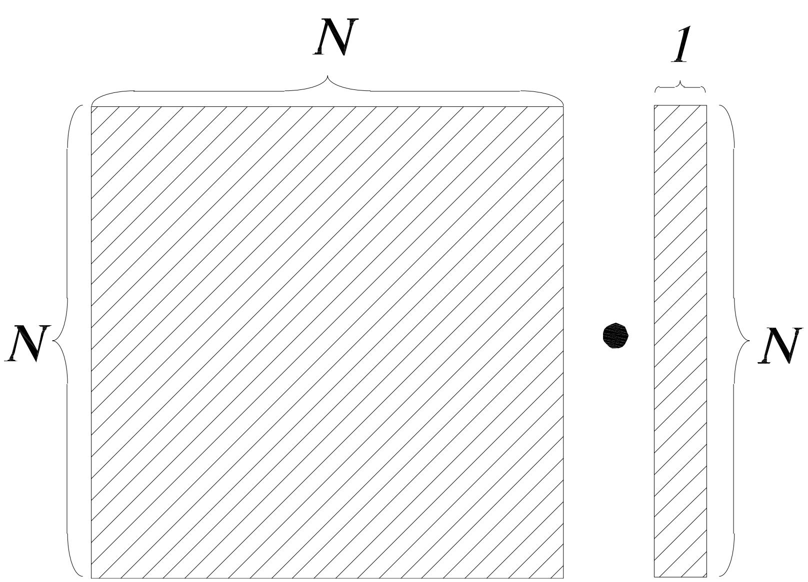
$$\frac{j_z(\vec{\rho})}{\sigma} + i\omega\mu_0 \iint_S G_0(\vec{\rho},\vec{\rho}\,')\, j_z(\vec{\rho}\,')\, ds' = -V_{p.u.l.},\ \vec{\rho} \in S$$

Static Green's function ➔

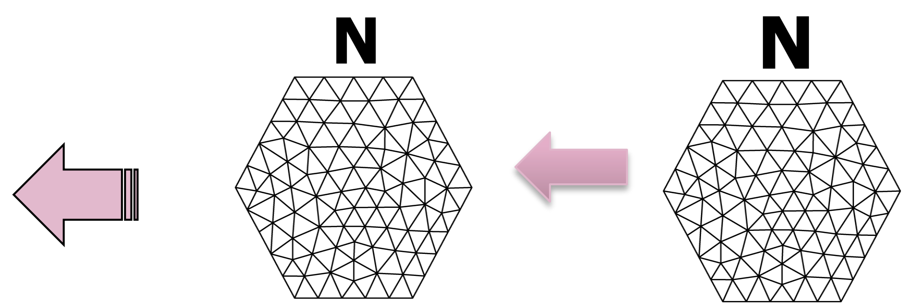
$$G_0(\vec{\rho},\vec{\rho}\,') = -\frac{1}{2\pi}\ln(|\,\vec{\rho}-\vec{\rho}\,'\,|)$$



Matrix-vector product:

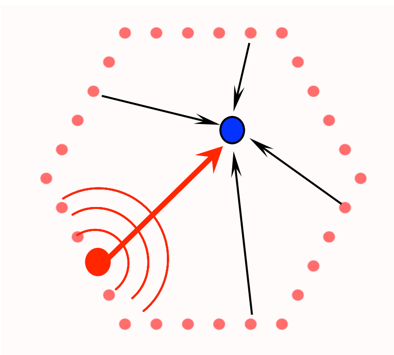


Volumetric mesh:



Surface-Volume-Surface EFIE

Volume EFIE ➔ $E_z(\vec{\rho}) + i\omega\mu_0\sigma \iint_S G_0(\vec{\rho},\vec{\rho}\,')E_z(\vec{\rho}\,')ds' = -V_{p.u.l.},\ \vec{\rho}\in S$



$$E_z(\vec{\rho}) = -i\omega\mu_0 \int_{\partial S} G_\sigma(\vec{\rho},\vec{\rho}\,')J_z(\vec{\rho}\,')d\vec{\rho}\,',\quad \vec{\rho}\in S$$

Full-wave Green's function ➔ $G_\sigma(\vec{\rho},\vec{\rho}\,') = \frac{i}{4}H_0^{(2)}\left(k_\sigma|\vec{\rho}-\vec{\rho}\,'|\right)$ $k_\sigma = \sqrt{\frac{\omega\mu_0\sigma}{2}}(1-i)$

Surface-Volume-Surface EFIE:

$$i\omega\mu_0 \int_{\partial S} G_\sigma(\vec{\rho},\vec{\rho}\,')J_z(\vec{\rho}\,')d\vec{\rho}\,' + \sigma(\omega\mu_0)^2 \int_{\partial S}\left[\iint_S G_0(\vec{\rho},\vec{\rho}\,')G_\sigma(\vec{\rho}\,',\vec{\rho}\,'')ds'\right]J_z(\vec{\rho}\,'')d\vec{\rho}\,'' = V_{p.u.l.}$$

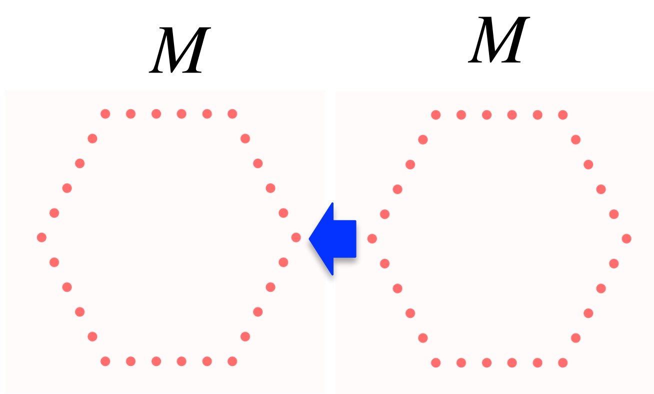
$$\vec{\rho}\in\partial S.$$

SVS-EFIE: Operators

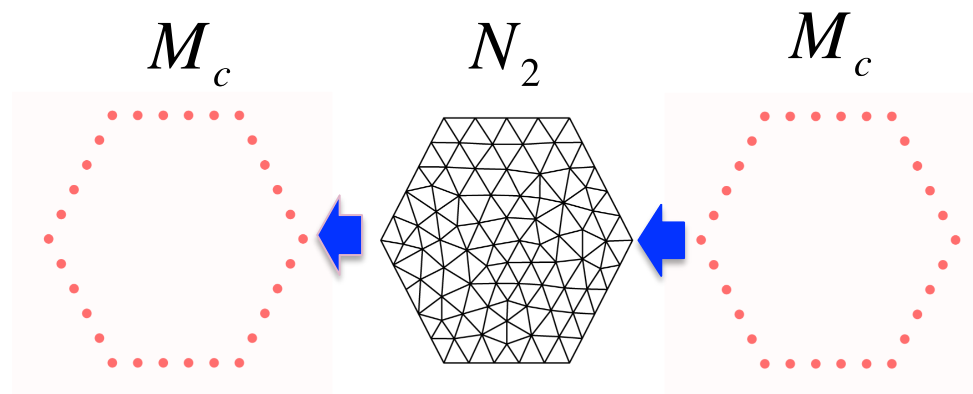
SVS-EFIE in operator form:

$$T_{\sigma}^{\partial S,\partial S} \circ J_z + \sigma\, T_0^{\partial S,S} \circ T_{\sigma}^{S,\partial S} \circ J_z = -V_{p.u.l.}$$

Surface-to-Surface mapping
(global impedance operator)



Surface-to-Volume-to-Surface mapping:



SVS-EFIE in matrix form:

$$(\mathbf{Z}_{\sigma}^{\partial S,\partial S} + \sigma \mathbf{Z}_0^{\partial S,S} \cdot \mathbf{Z}_{\sigma}^{S,\partial S}) \cdot \mathbf{I} = \mathbf{V}$$

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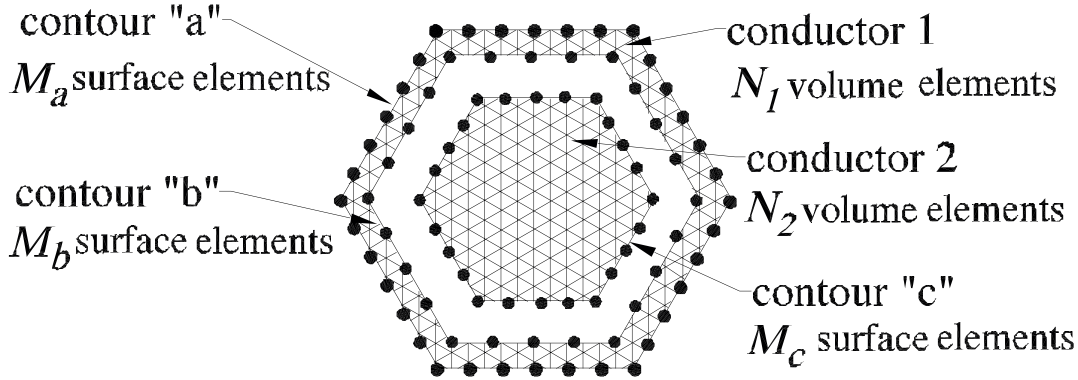
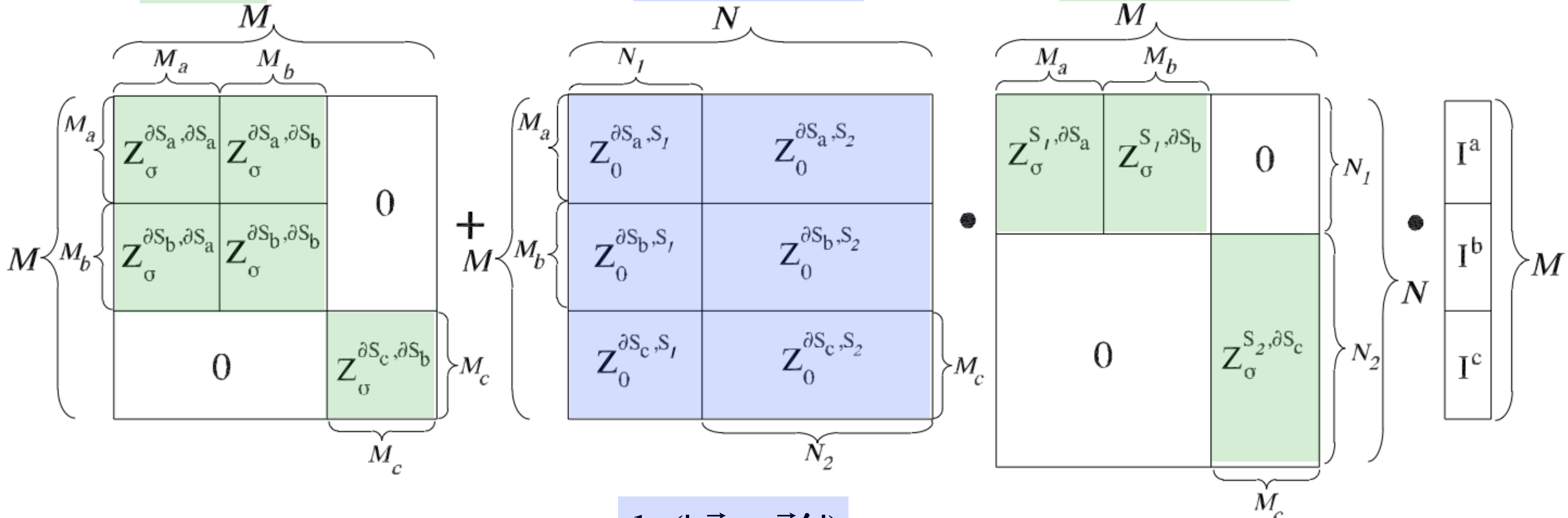
SVS-EFIE: Matrix structure



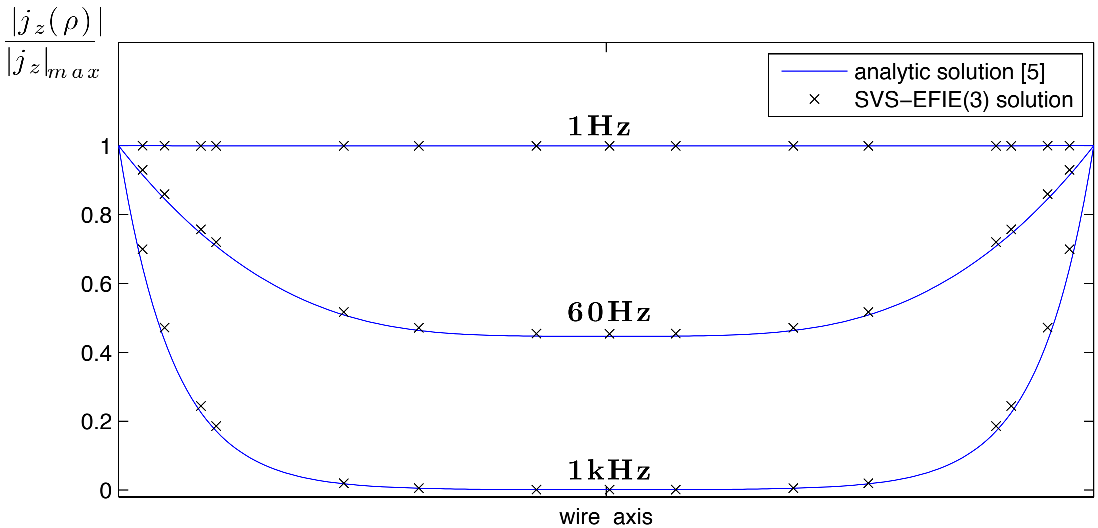
$$\left(Z_\sigma^{\partial S,\partial S} + \sigma Z_0^{\partial S,S} \cdot Z_\sigma^{S,\partial S} \right) \cdot I = V$$



$$H_0^{(2)}\left(k_\sigma\left|\vec{\rho}-\vec{\rho}\,'\right|\right) \qquad \ln(|\vec{\rho}-\vec{\rho}\,'|) \qquad H_0^{(2)}\left(k_\sigma\left|\vec{\rho}-\vec{\rho}\,'\right|\right)$$

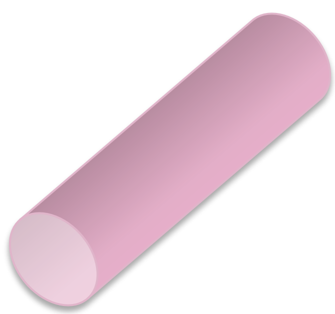
SVS-EFIE vs. analytic solution

Comparison of new model to the analytic solution for ideal circular cross-section



$$\left(\frac{|j_z(\rho)|}{|j_z|_{\max}}\right)_{analytic} = \frac{J_0(k_\sigma \cdot r)}{J_0(k_\sigma \cdot r_0)}$$

where:
J_0 is Bessel function,
r – radial coordinate
r_0 – radius of cross-section



Max Relative error < 2.5%

Simulation Parameters	
Transmission line	Single aluminum conductor, circular cross-section, r0=0.025m
Surface mesh	200 linear elements
Volumetric mesh	3,620 triangular elements

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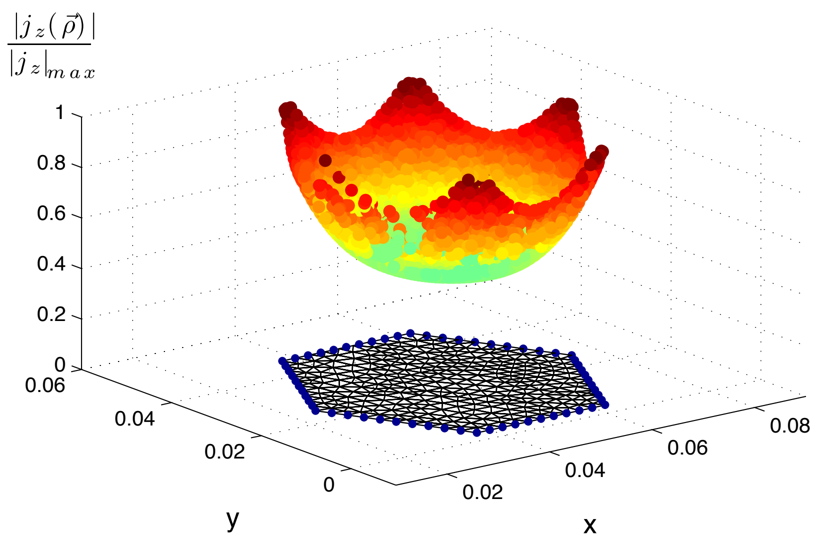
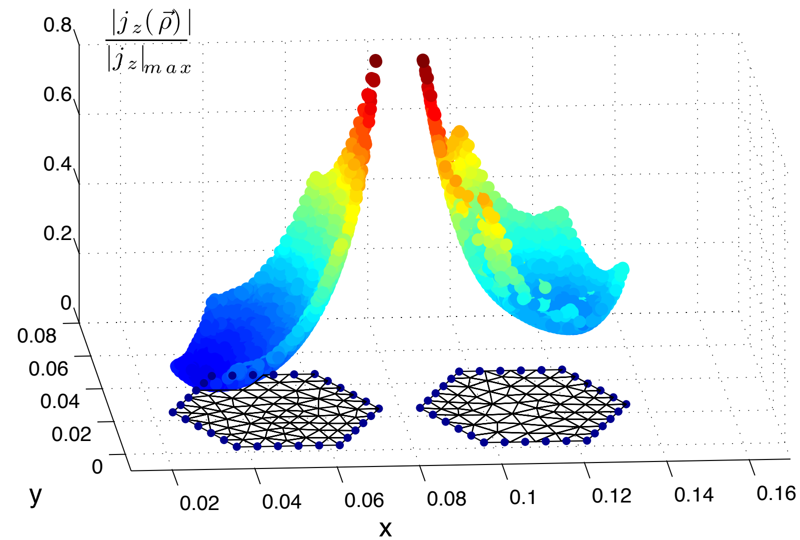
SVS-EFIE vs V-EFIE: Example 1



Relative error[1] < 2.7%

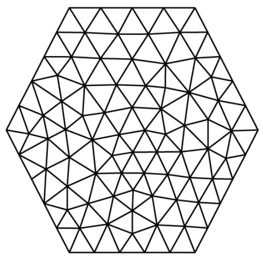


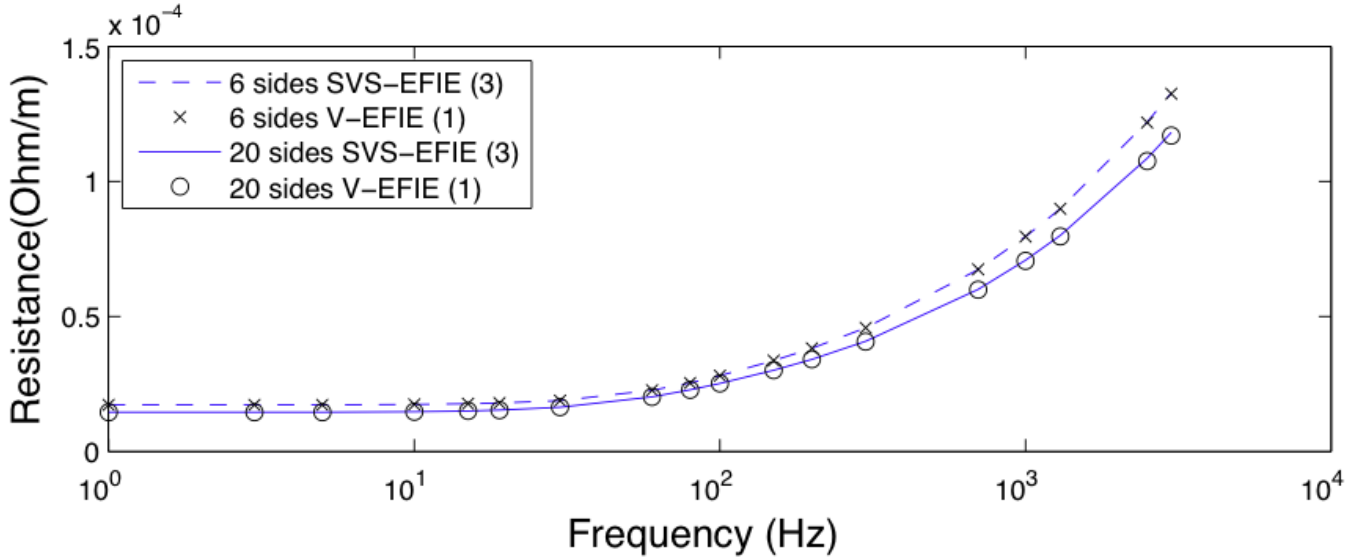
Relative error[1] < 1.8%

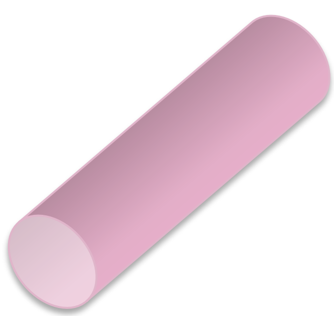
Simulation Parameters		
Transmission line	Single aluminum conductor, hexagonal cross-section, r0=0.025m	2 aluminum conductors of hexagonal cross-section, r0=0.025, centers separated by 0.06m, left – grounded, right – driven 1 V/m
Frequency	60 Hz	60 Hz
Surface mesh	60 linear elements	2×60=120 linear elements
Volumetric mesh	3,697 triangular elements	3,094 triangular elements

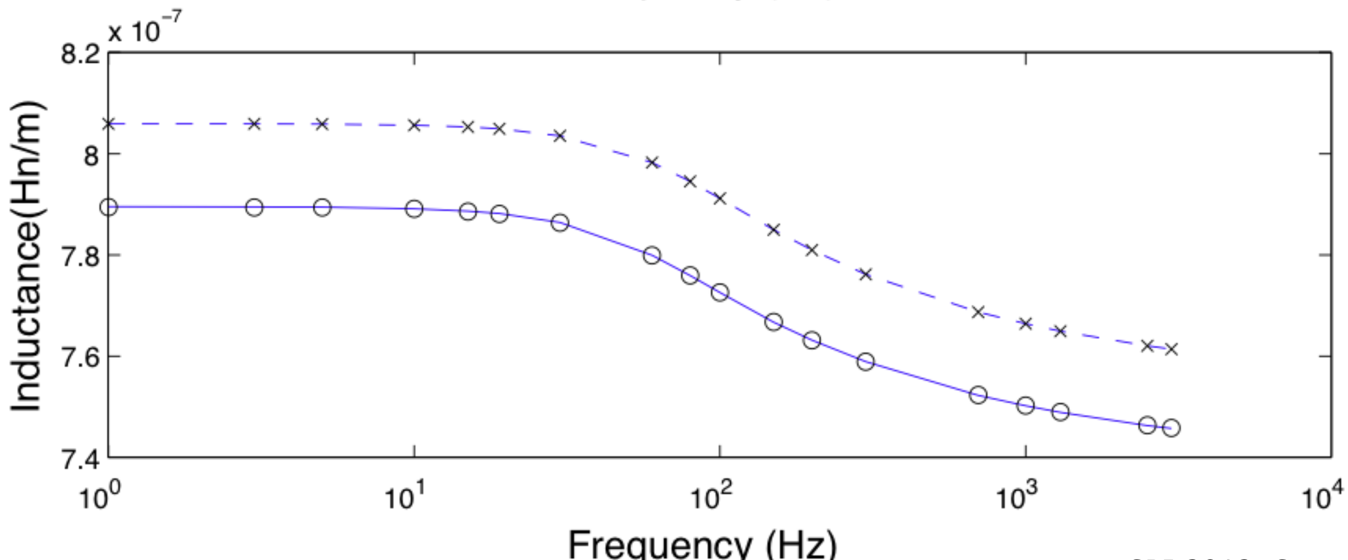
[1] – with respect to traditional Volume-EFIE solution

R and L for circular conductor

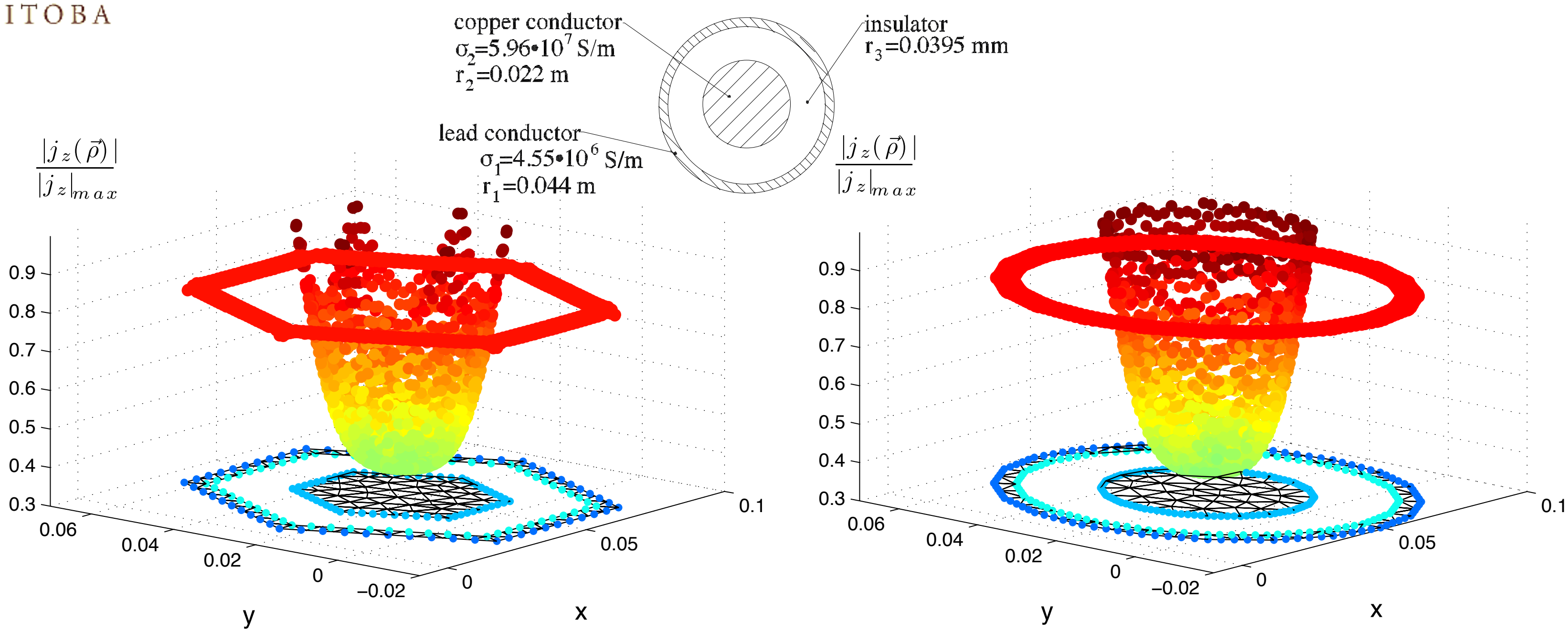








SVS-EFIE vs V-EFIE: Example 2

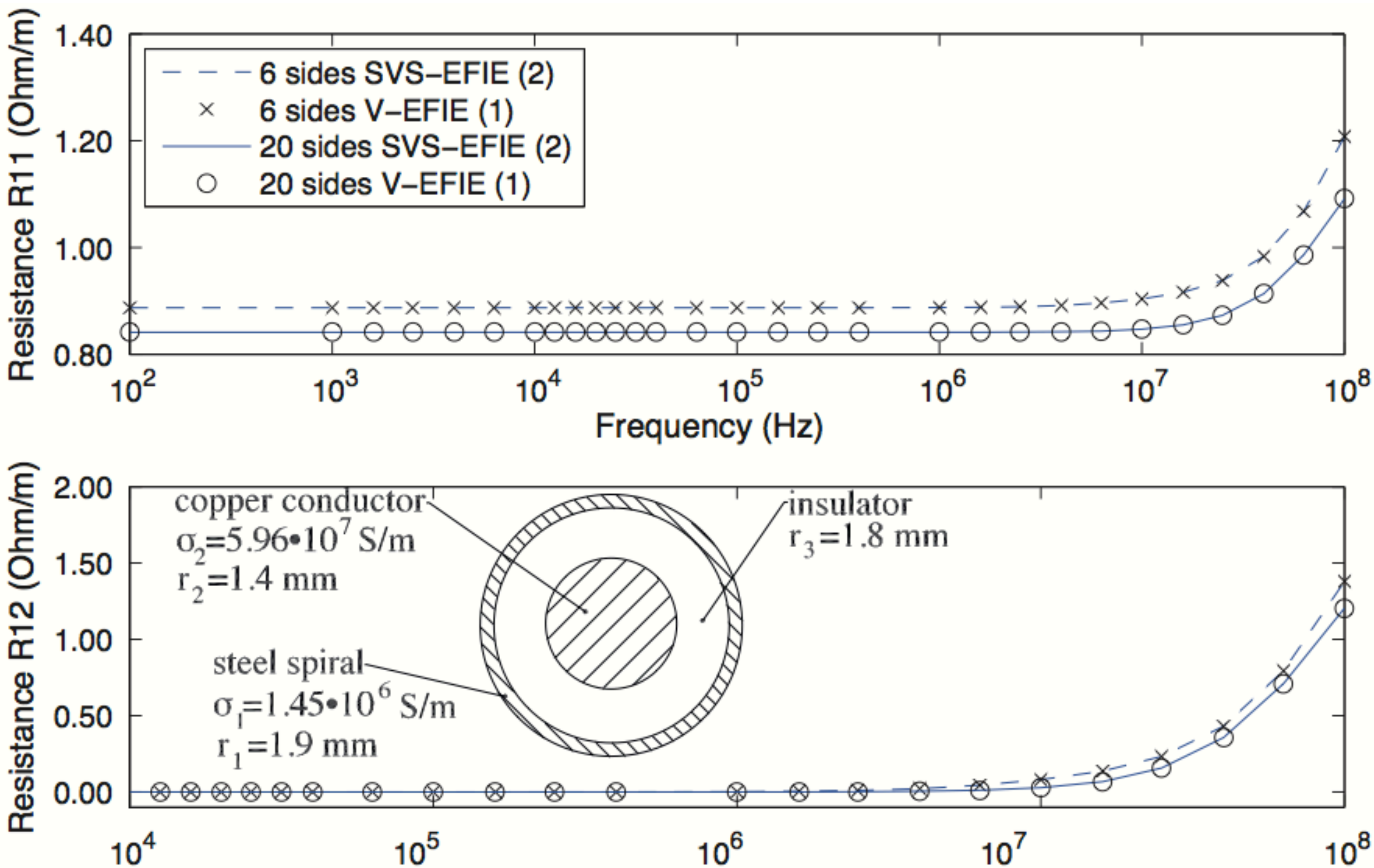


Relative error[1] < 0.5% **Relative error[1] < 0.5%**

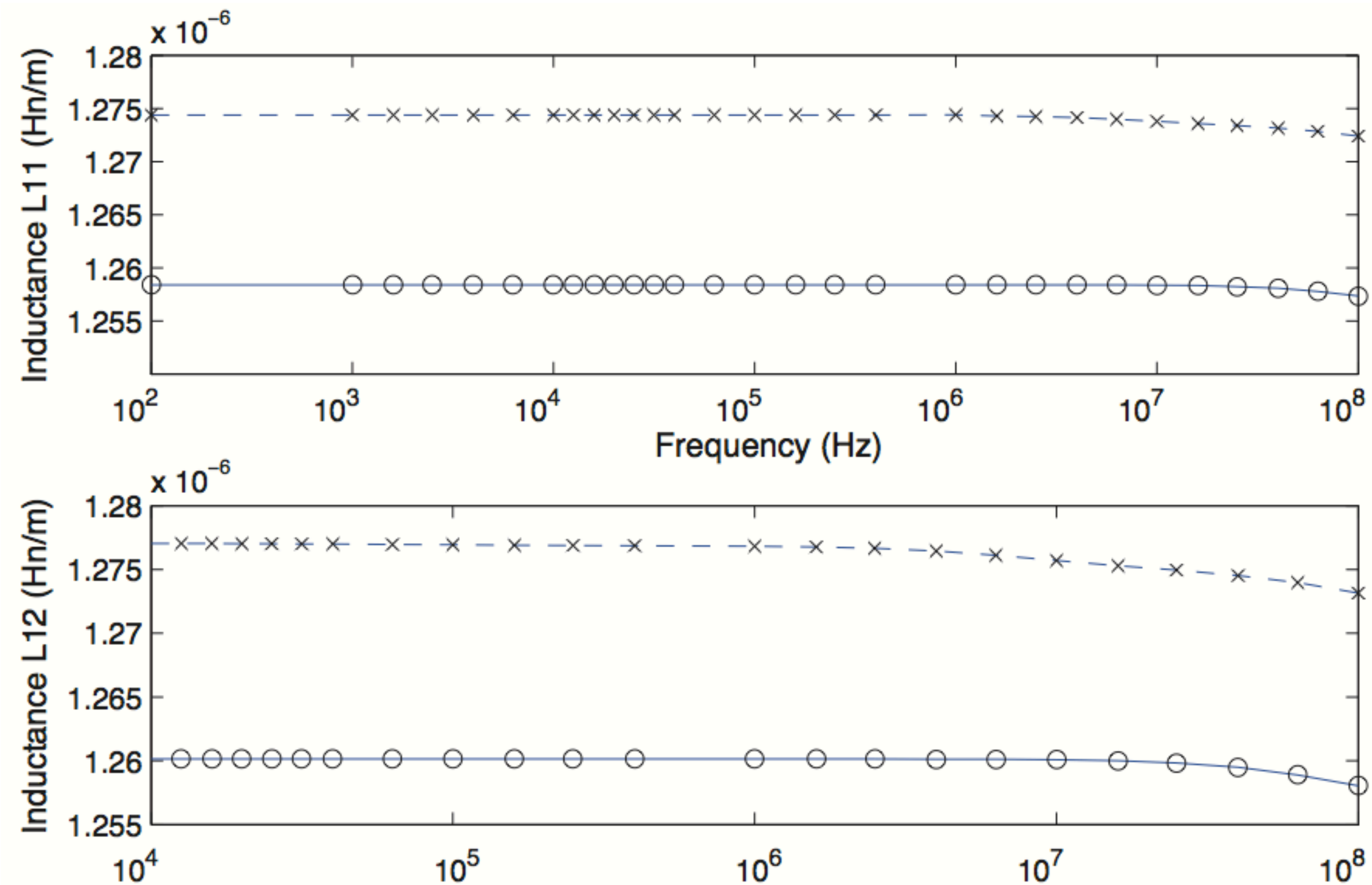
Simulation Parameters		
Cross-section approximation	three hexagons	three 20-sided polygons (icosagons)
Mode	60 Hz, inner conductor is grounded, outer is driven 1 V/m	
Surface mesh	3•60=120 linear elements	3•100=3,000 linear elements
Volumetric mesh	1,660 triangular elements	1,376 triangular elements

1 – with respect to traditional Volume-EFIE solution

Resistance for coaxial cable



Inductance for coaxial cable



Conclusions

- Surface-Volume-Surface EFIE:
- Pros:
  - Degrees of freedom are surface based
  - Degrees of freedom are independent on frequency
  - Rigorous formulation
    - No loss of accuracy from approximations
  - No derivatives on the Green's function
    - Easily combines with layered media Green's function
  - Highly sparse matrices at high frequencies
- Cons:
  - Requires both surface and volume meshing

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Questions?