UNIVERSITEIT GENT

FACULTY OF ENGINEERING

PARAMETERIZED REDUCED ORDER MODELS WITH GUARANTEED PASSIVITY USING MATRIX INTERPOLATION

Elizabeth Rita Samuel
Elizabeth_rita.samuel@intec.ugent.be
www.ibcn.intec.ugent.be
Internet Based Communication Networks and Services (IBCN)
Department of Information Technology (INTEC)
Ghent University - IBBT

Authors: Elizabeth Rita Samuel, Francesco Ferranti, Luc Knockaert, and Tom Dhaene

OVERVIEW

- **Introduction**
- **Proposed method**
- **Numerical example**
- **Results**
- **Conclusions**

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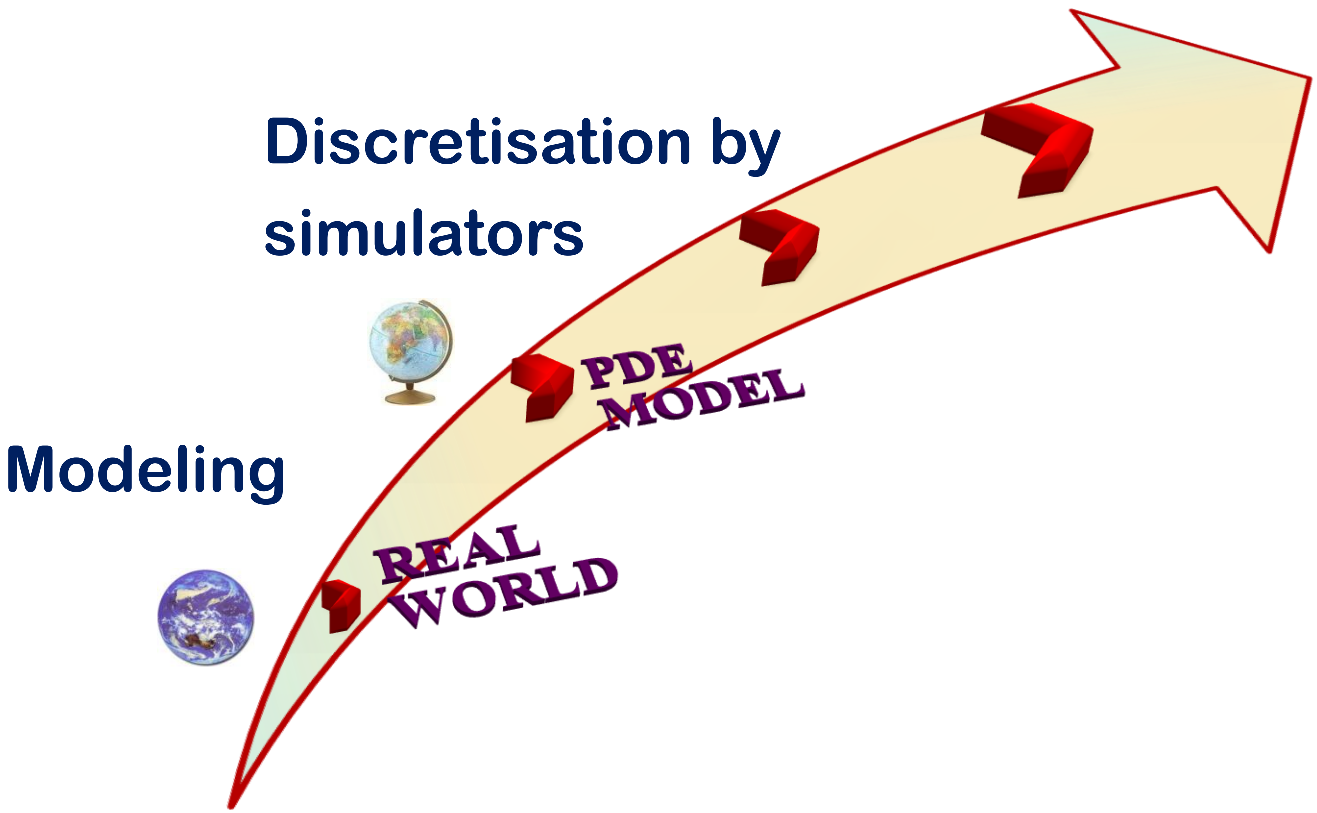

Discretisation by simulators
PDE MODEL
Modeling
REAL WORLD

INTRODUCTION

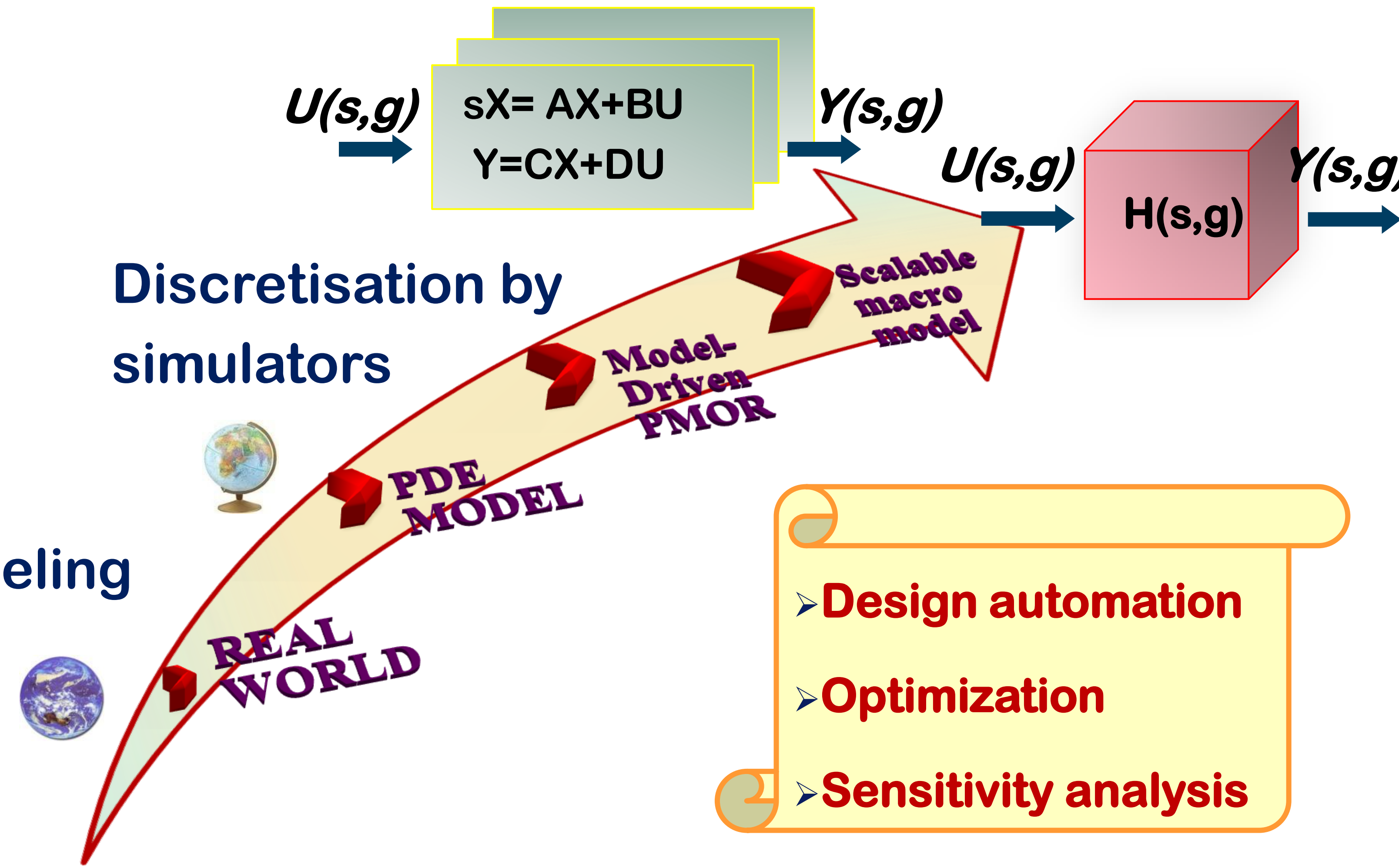
$U(s,g) \rightarrow$ $sX = AX + BU$, $Y = CX + DU$ $\rightarrow Y(s,g)$

Discretisation by simulators

Modeling

- Environmental effects
- Manufacturing variations
- Critical dimension fluctuations

INTRODUCTION

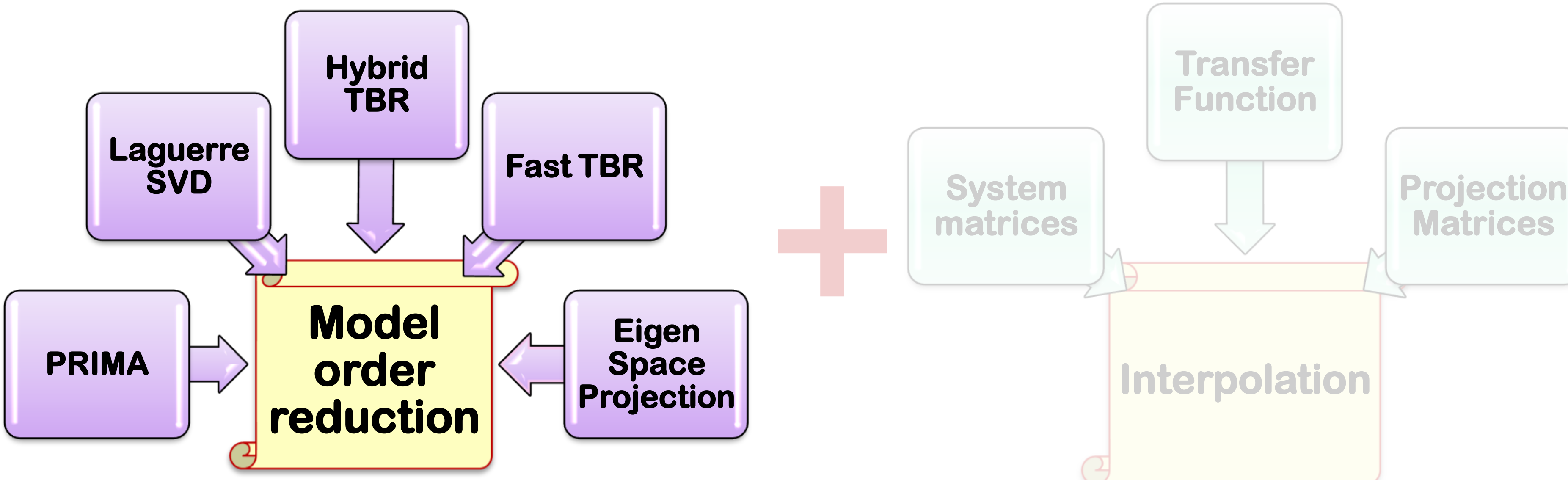


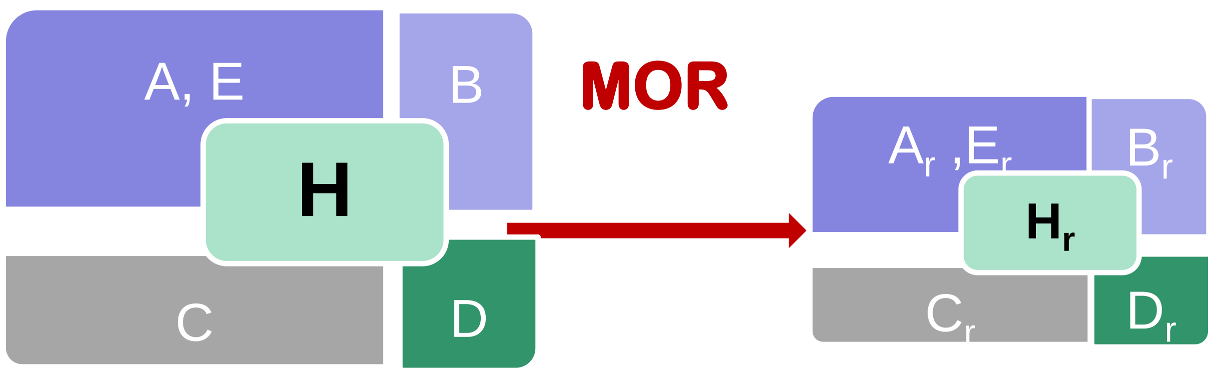
Discretisation by simulators

Modeling

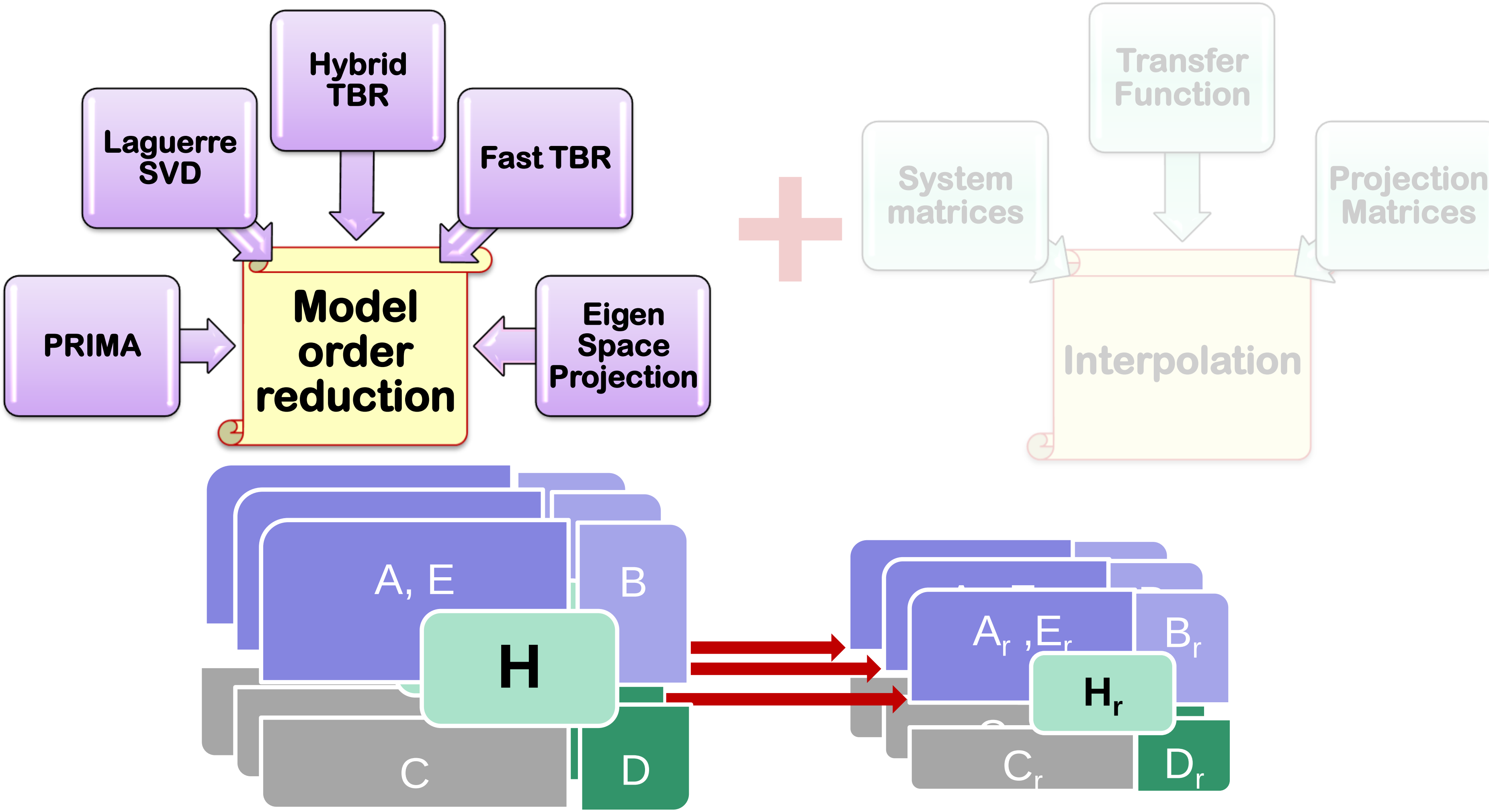
- Design automation
- Optimization
- Sensitivity analysis

INTRODUCTION





INTRODUCTION



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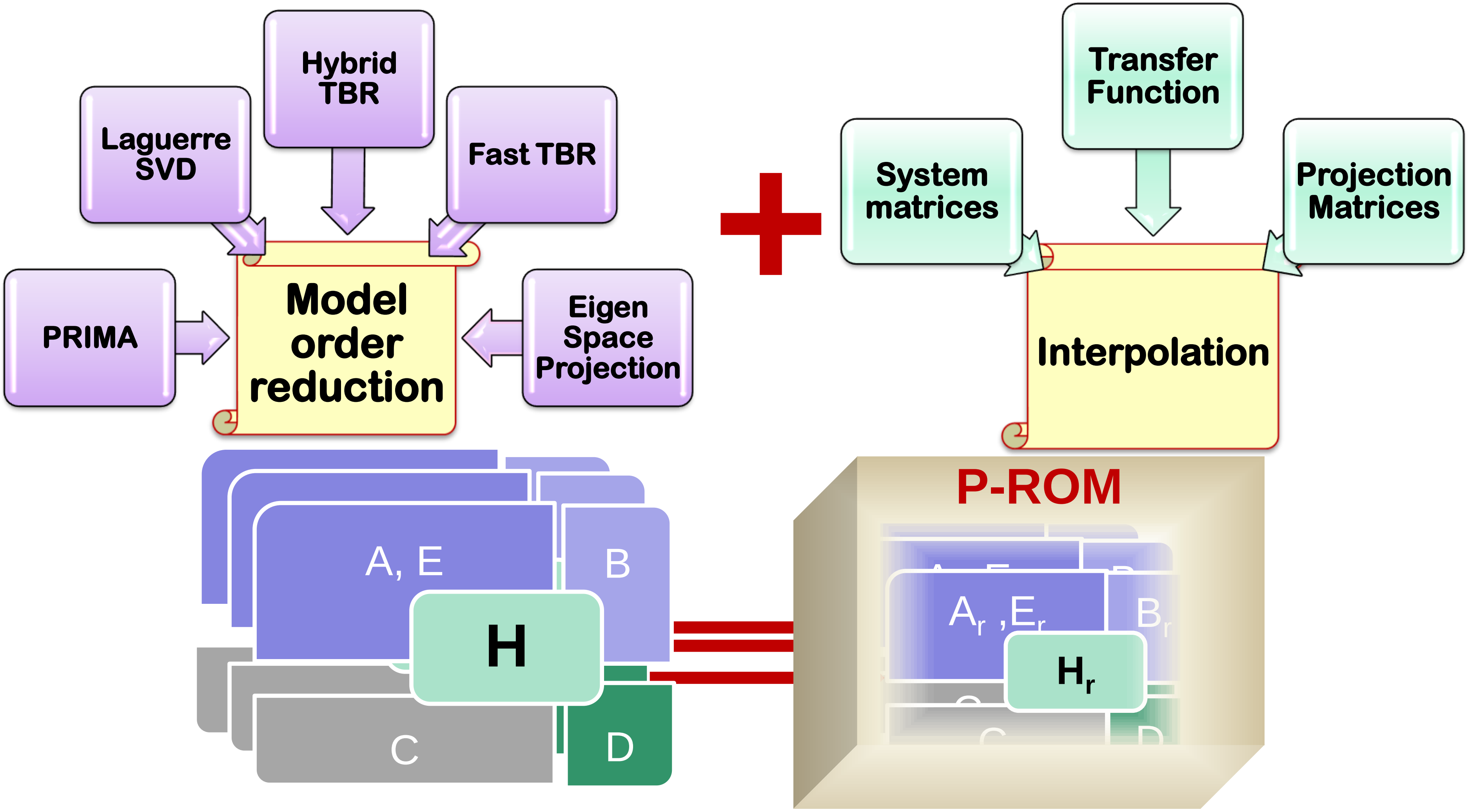
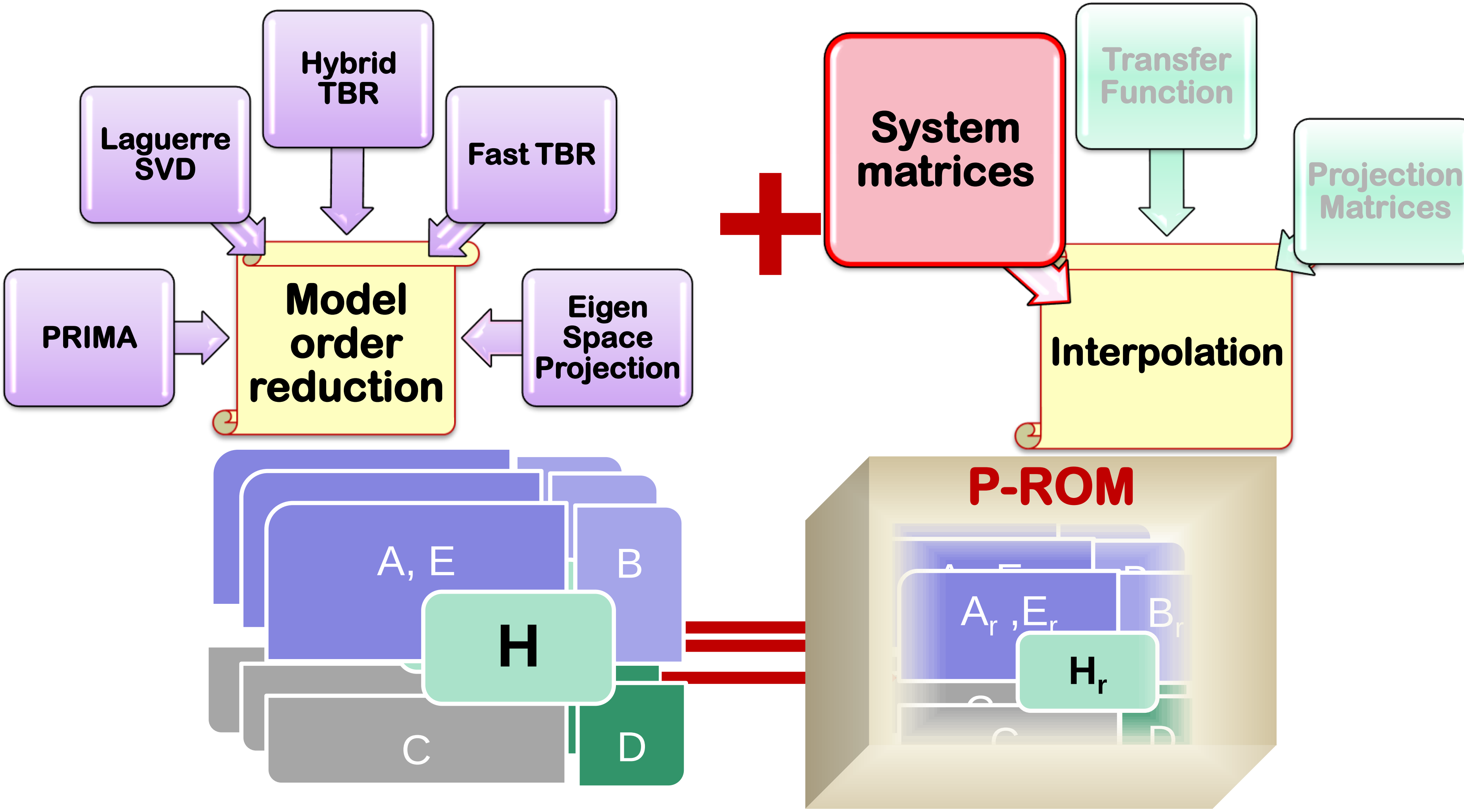
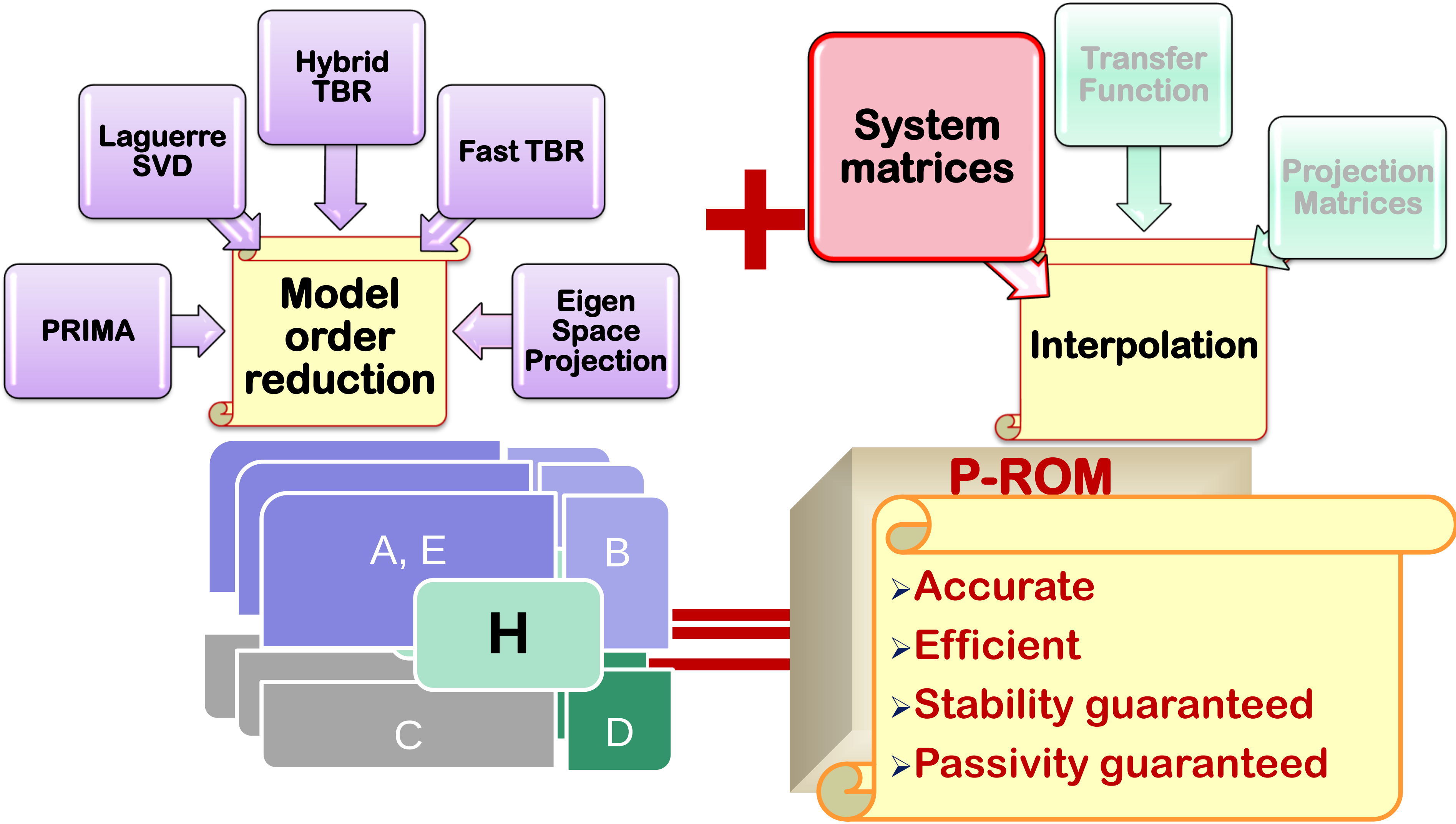


INTRODUCTION



INTRODUCTION



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TECHNIQUE OF PANZER et al.

Compute ROM

Modified ROM

Interpolate the modified ROM to obtain the P-ROM

* H. Panzer, J. Mohring, R. Eid, and B. Lohmann, “Parametric model order reduction by matrixinterpolation,” Automatisierungstechnik,pp. 475–484, Aug. 2010.

TECHNIQUE OF PANZER et al.

Compute ROM

Modified ROM

Interpolate the modified ROM to obtain the P-ROM

$$E\dot{x}(t,g) = Ax(t,g) + Bu(t),$$
$$y(t,g) = Cx(t,g) + Du(t)$$

V, W

$$E_r\dot{z}(t,g) = A_r z(t,g) + B_r u(t),$$
$$y(t,g) = C_r z(t,g) + D_r u(t)$$
$$where$$
$$E_r = W^T EV, A_r = W^T AV$$
$$B_r = W^T B, C_r = CV$$

TECHNIQUE OF PANZER et al.

Compute ROM

Modified ROM

Interpolate the modified ROM to obtain the P-ROM

TRANSFORMATION

Using matrices

$\mathbf{T_i} \in \mathbf{R}^{qxq}$

where $\mathbf{T_i = R^T V_i}$

$\mathbf{M_i} \in \mathbf{R}^{qxq}$

where $\mathbf{M_i = W_i^T R}$

BACK PROJECTION

Using Matrices

$\mathbf{V_i} \in \mathbf{R}^{nxq}$

$\mathbf{Wi} \in \mathbf{R}^{nxq}$

REPROJECTION

Using an orthogonal matrix

R

$V_{all} = [\, V_1 \; V_2 \ldots]$

$V_{all} = \mathbf{U} \Sigma N^T$

R is the first q columns of **U**

TECHNIQUE OF PANZER et al.

Compute ROM

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Interpolate the modified ROM to obtain the P-ROM

PROPOSED TECHNIQUE

➢ PMOR using Matrix interpolation

- The proposed technique enhances the technique of Panzer et al.

- **PMOR using Matrix interpolation**
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    - **Automatized estimation of the reduced order.**

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  - **Automatized estimation of the reduced order.**
  - **By generating a compact common projection matrix for congruence transformation.**

PROPOSED TECHNIQUE

➢ PMOR using Matrix interpolation

- The proposed technique enhances the technique of Panzer et al.
  - **Automatized estimation of the reduced order.**
  - **By generating a compact common projection matrix for congruence transformation.**
  - **By using passivity-preserving parameterization schemes.**

PROPOSED TECHNIQUE

Estimate the reduced order → Compute the common projection matrix → Perform congruence transformation → Positive Interpolation

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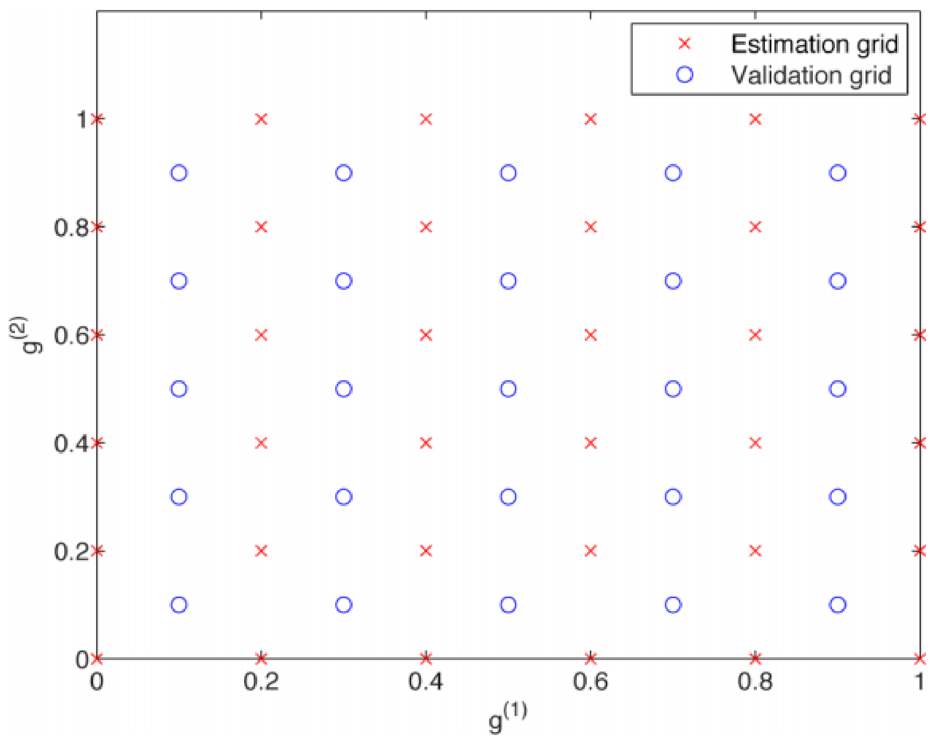


- Estimate the reduced order at the corner points of the design space.
- Reduced order is chosen in two ways
  - **Worst-case reduced order**
  - **Best-case reduced order**

PROPOSED TECHNIQUE

Estimate the reduced order → **Compute the common projection matrix** → Perform congruence transformation → Positive Interpolation

Starting from a set of models generated from a common space

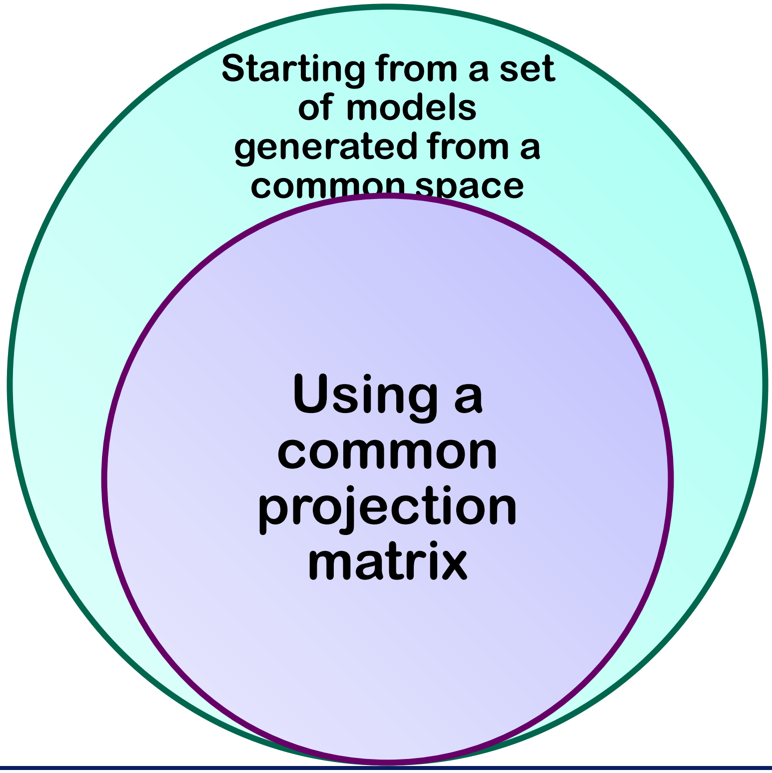
PROPOSED TECHNIQUE

Estimate the reduced order

Compute the common projection matrix

Perform congruence transformation

Positive Interpolation



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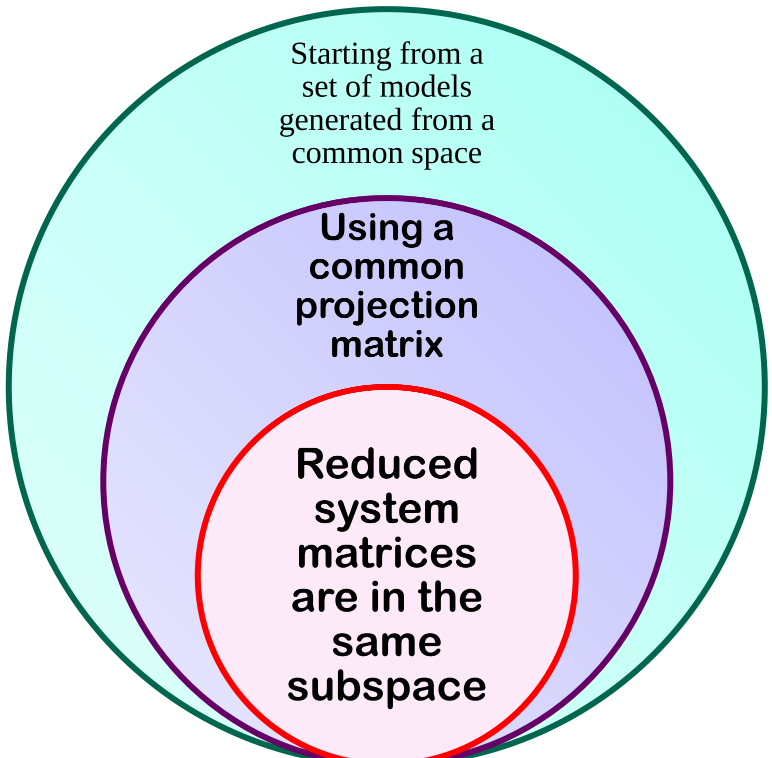

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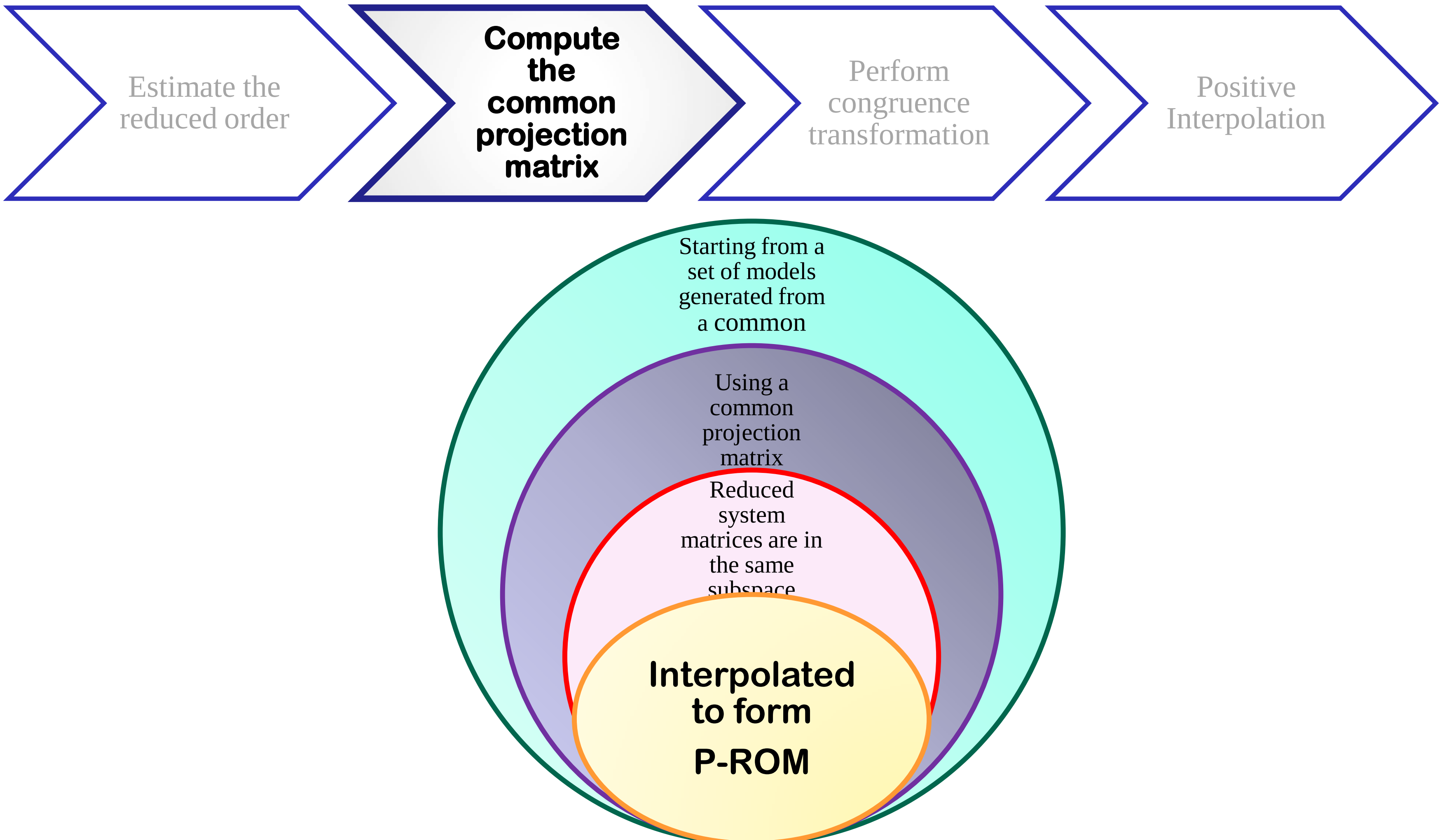


PROPOSED TECHNIQUE

Estimate the reduced order

Compute the common projection matrix

Perform congruence transformation

Positive Interpolation

A Krylov based MOR technique is used to compute the projection matrices

- **Locally or Globally**

PROPOSED TECHNIQUE

Estimate the reduced order

Compute the common projection matrix

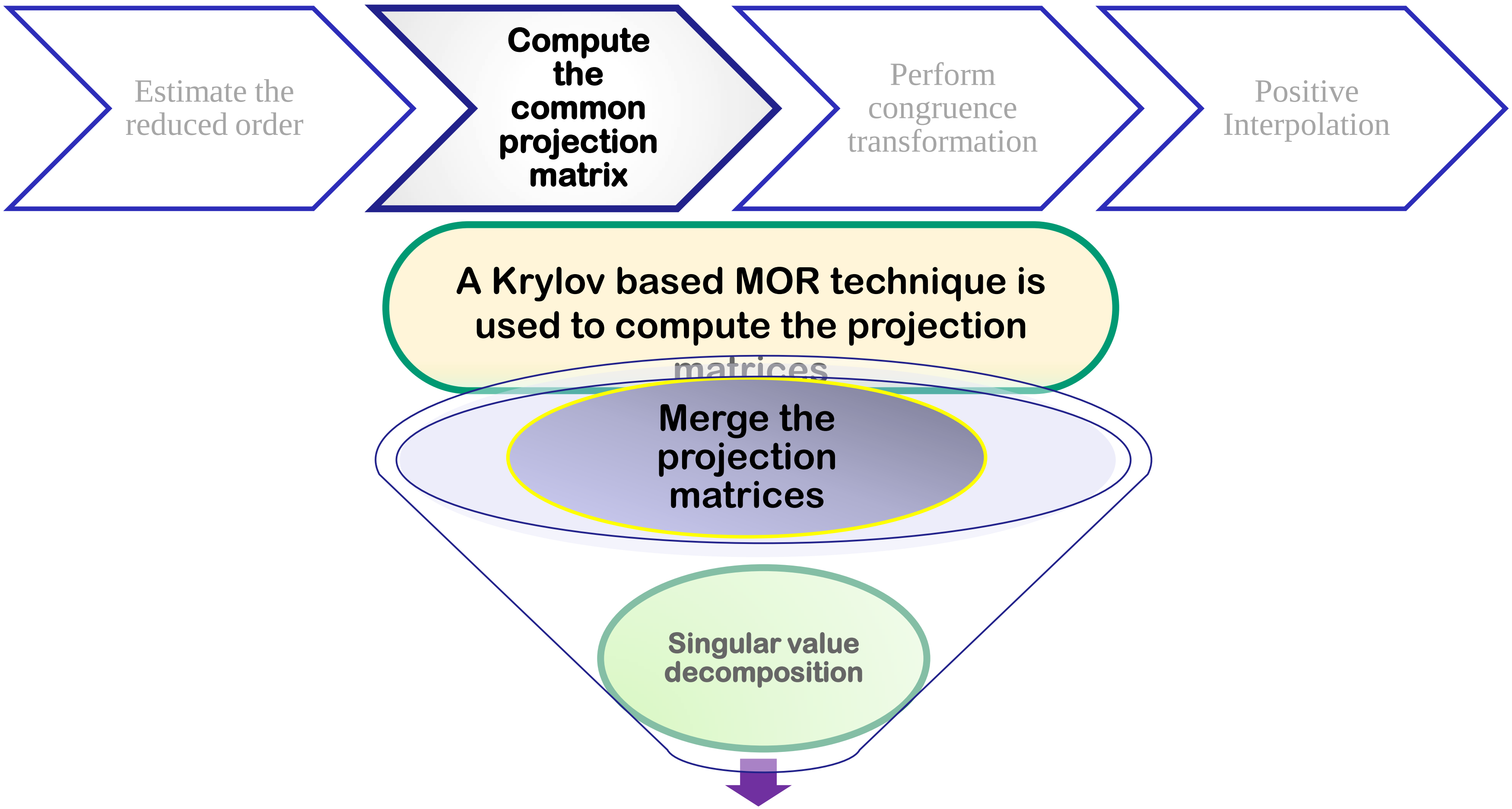
Perform congruence transformation

Positive Interpolation

A Krylov based MOR technique is used to compute the projection matrices

Merge the projection matrices

PROPOSED TECHNIQUE



Compact common projection matrices, Qcomm

PROPOSED TECHNIQUE

Estimate the reduced order

Compute the common projection matrix

Perform congruence transformation

Positive Interpolation

$$C(g_k)\dot{x}(t,g_k) = -G(g_k)x(t,g_k) + Bu(t,g_k)$$

$$y(t,g_k) = L^T x(t,g_k) + Du(t,g_k)$$

satisfies

$$C(g_k) = C(g_k)^T \geq 0$$

$$G(g_k) + G(g_k)^T \geq 0$$

$$B = L$$

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Estimate the reduced order → Compute the common projection matrix → **Perform congruence transformation** → Positive Interpolation

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$$B = L$$

PROPOSED TECHNIQUE

Estimate the reduced order → Compute the common projection matrix → **Perform congruence transformation** → Positive Interpolation

$$C_r(g_k) = Q_{comm}^T C(g_k) Q_{comm} \geq 0$$

$$G_r(g_k) = Q_{comm}^T G(g_k) Q_{comm} \geq 0$$

$$B_r = Q_{comm}^T B$$

$$L_r = Q_{comm}^T L$$

$$C_r(g_k)\dot{z}(t,g_k) = -G_r(g_k) z(t,g_k) + B_r u(t,g_k)$$

$$y_r(t,g_k) = L_r{}^T z(t,g_k) + D_r u(t,g_k)$$

Estimate the reduced order

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Positive Interpolation

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PROPOSED TECHNIQUE

Estimate the reduced order → Compute the common projection matrix → Perform congruence transformation → **Positive Interpolation**

- **Positive Interpolation is used to preserve the overall passivity.**
- **E.g, for multilinear interpolation:**

$$T(g^{(1)},\dots,g^{(N)}) = \Sigma_{k_1=1}^{K_1} \dots \Sigma_{k_N=1}^{K_1} T_{(g_{k_1}^{(1)},\dots,g_{kN}^{(N)})} l_{k_1}(g^{(1)}) \dots . l_{k_N}(g^{(N)})$$

where

$l_{k_i}(g^{(i)})$ *are the piecewise interpolation kernel*

$$0 \le l_{k_i}(g^{(i)}) \le 1$$

$$\sum_{k=1}^{K_1} l_{k_i}(g^{(i)}) = 1$$

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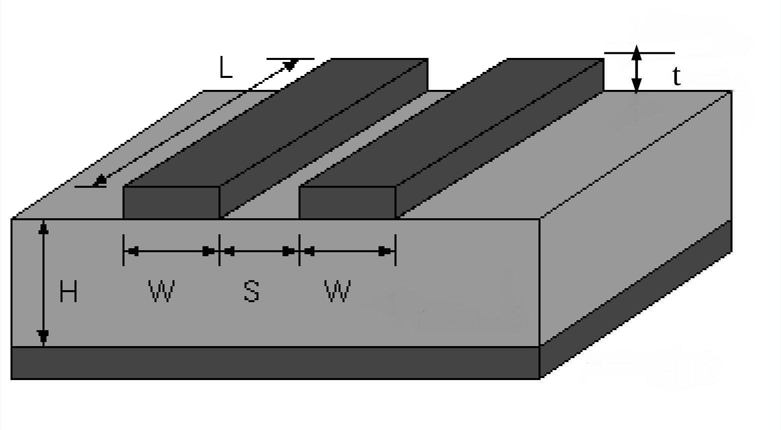
NUMERICAL EXAMPLE

- A coupled microstrip*
  - Width, w = 100µm
  - Thickness, t =50 µm

Parameter	Range	
	Min	Max
Frequency	1KHz	4 GHz
Length (*L*)	2 cm	6 cm



** L. Knockaert and D. De Zutter, "Laguerre-SVD reduced order modeling," IEEE Transactions on Microwave Theory and Techniques, vol. 48, no. 9, pp. 1469 –1475, sep.2000.*

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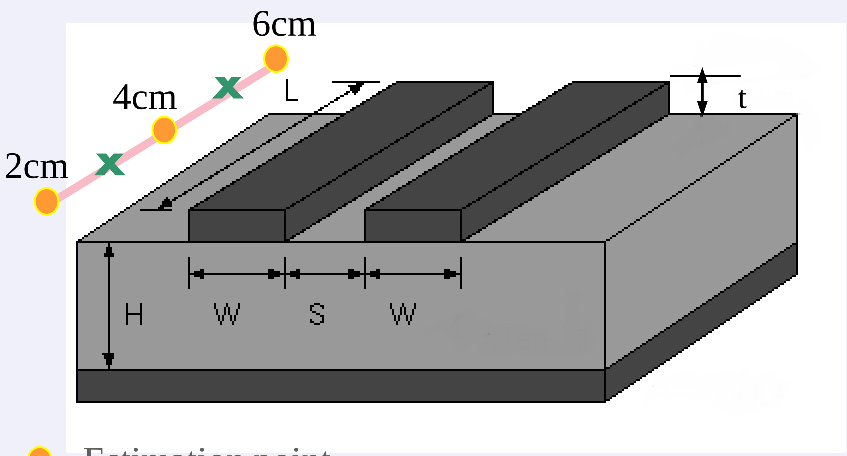

- Estimation point

x - Validation point

Parameter	No. of samples generated	No. of Samples for modeling
Length (*L*)	5	**3**

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RESULTS

Projection Matrices

Threshold for truncating the singular values, σ

Weighted RMS Error for Accuracy check

RESULTS

Projection Matrices → Laguerre-SVD

Threshold for truncating the singular values, σ

Weighted RMS Error for Accuracy check

RESULTS

Projection Matrices → Laguerre-SVD

Threshold for truncating the singular values, σ → $$\frac{\sigma_i}{\sigma_{\max}} \geq 0.01$$

Weighted RMS Error for Accuracy check

RESULTS

Projection Matrices → **Laguerre-SVD**

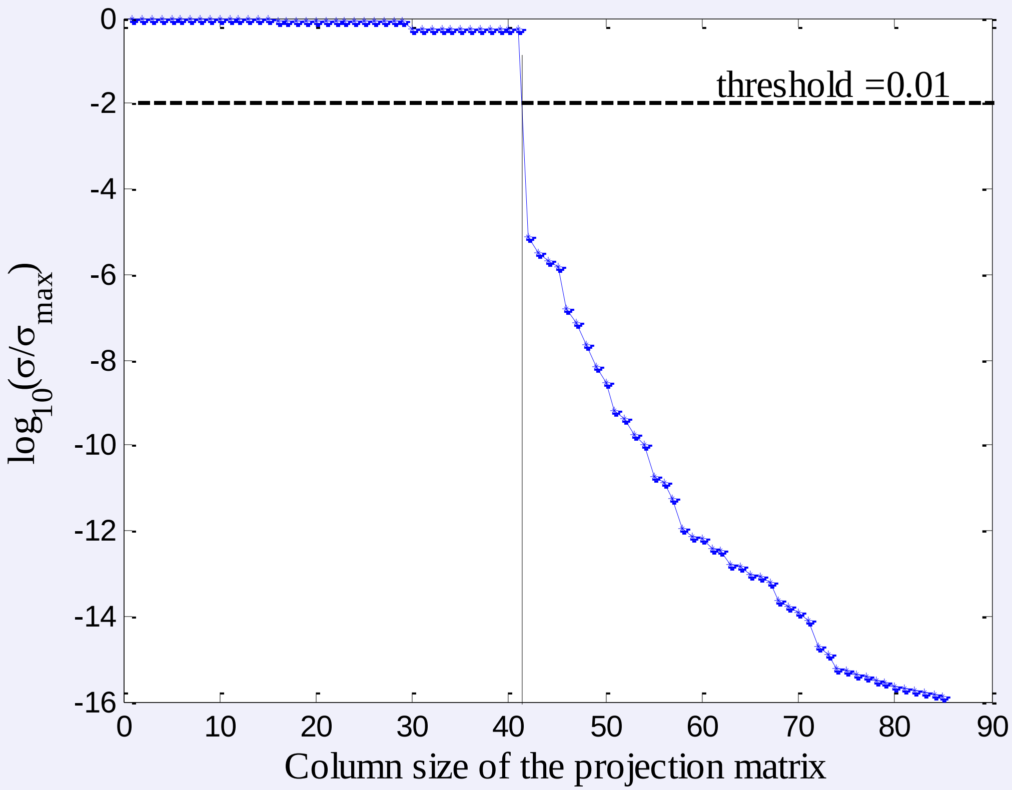
Threshold for truncating the singular values, σ →

$$\frac{\sigma_i}{\sigma_{\max}} \geq 0.01$$

Weighted RMS Error for Accuracy check →

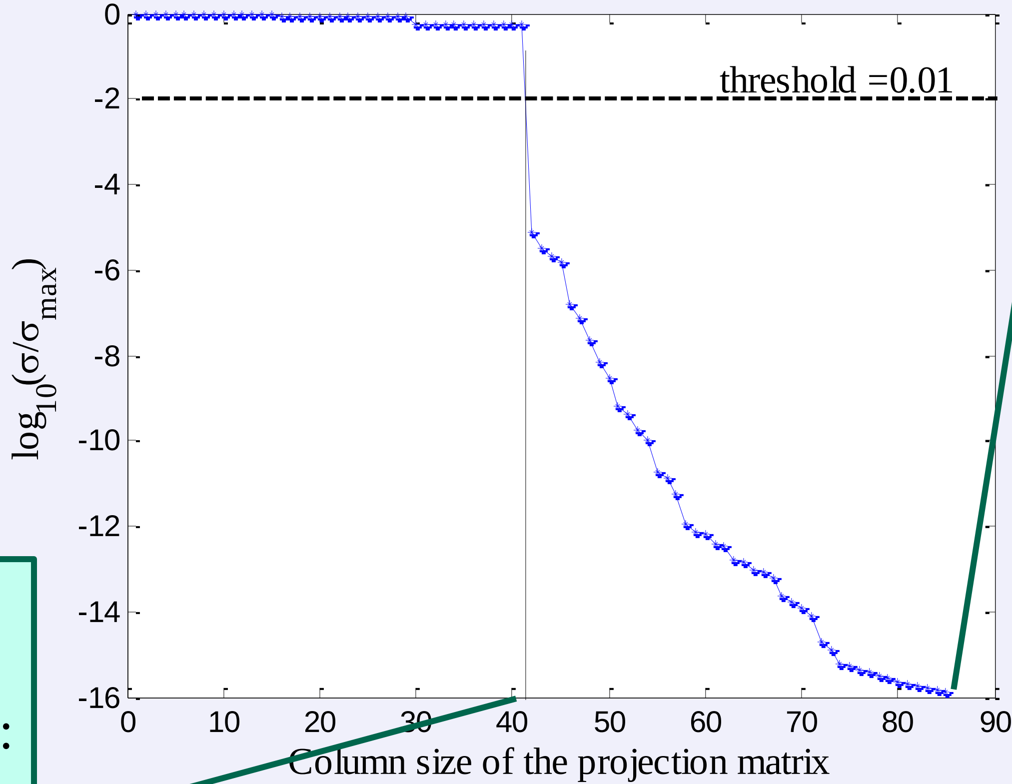
$$Err(Y_1(s), Y_2(s)) = \sqrt{\frac{\sum_{k=1}^{K_s}\sum_{i=1}^{P_{in}}\sum_{j=1}^{P_{out}} \frac{\left|Y_{1,(ij)}(s_k) - Y_{2,(ij)}(s_k)\right|^2}{\left|Y_{2,(ij)}(s_k)\right|^2}}{K_s P_{in} P_{out}}}$$

Singular values of the projection matrix



RESULTS

Singular values of the projection matrix

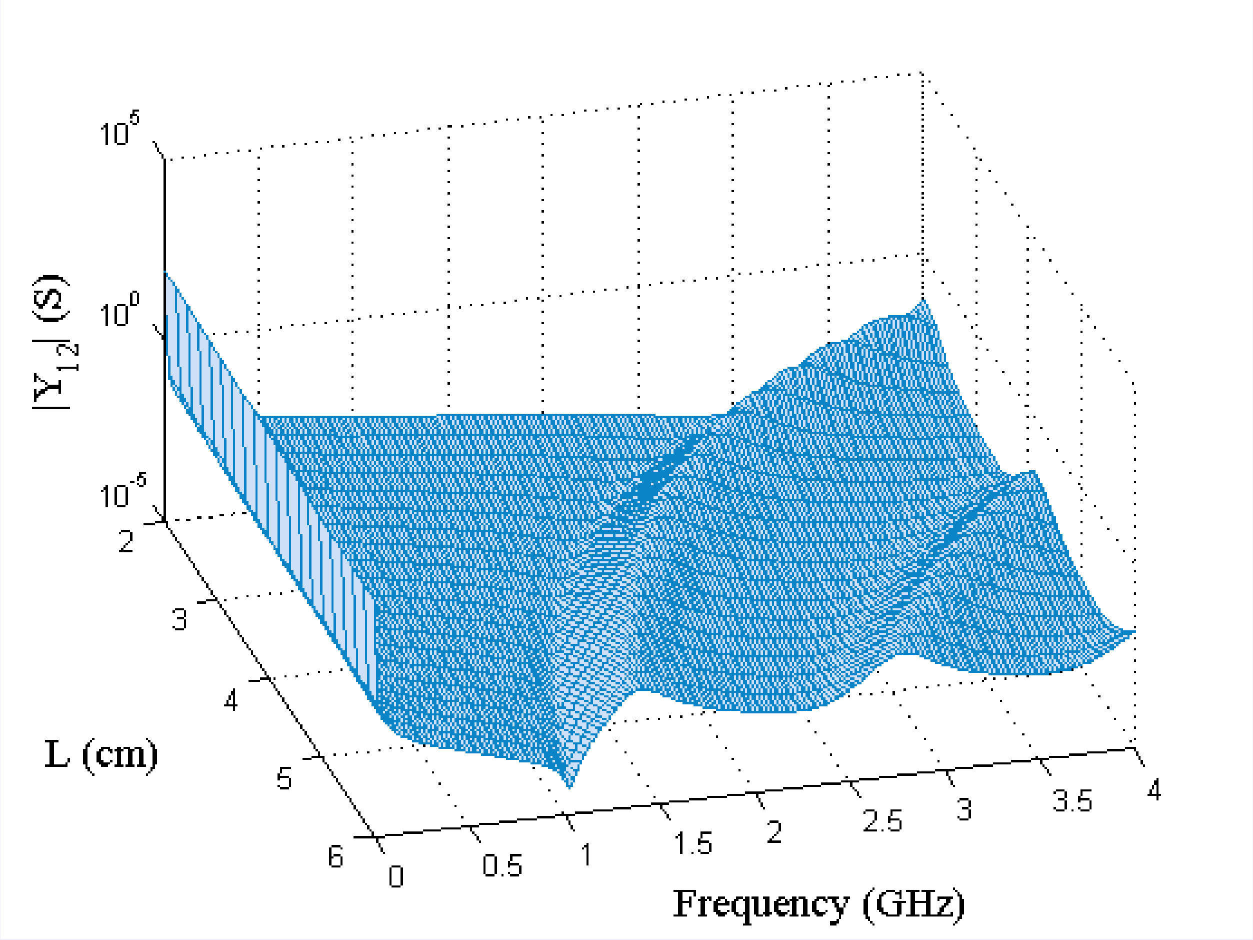


Column size of the merged projection matrix: 85

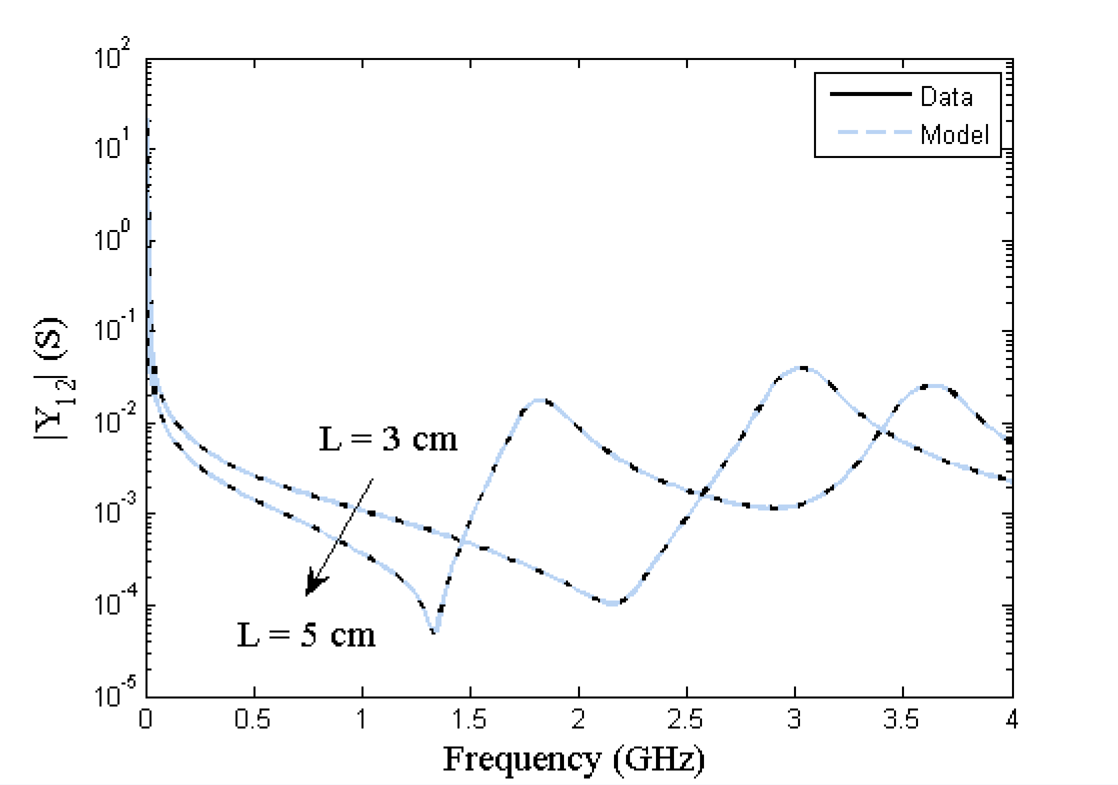
Column size of the common projection matrix: 42

RESULTS

Magnitude of bivariate reduced model $Y_{12}(s,L)$



Magnitude of the bivariate reduced model $Y_{12}(s,L)$ for L={3,5}cm using a common projection matrix



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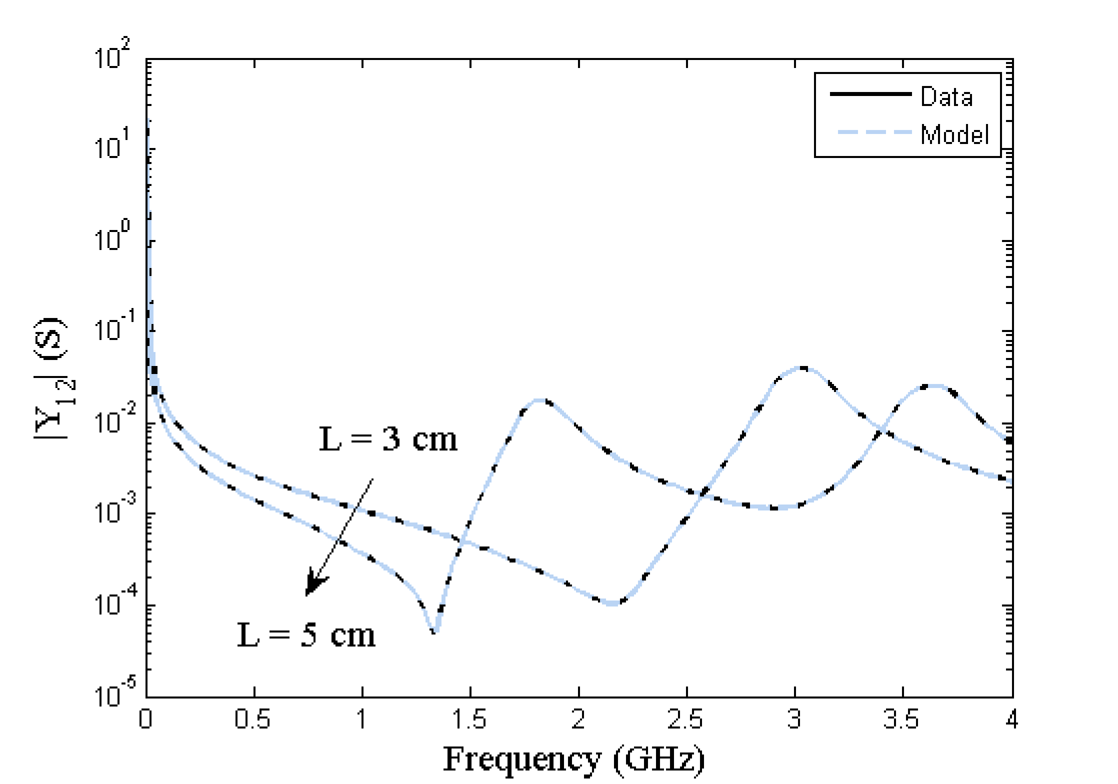

Weighted RMS error =0.0196

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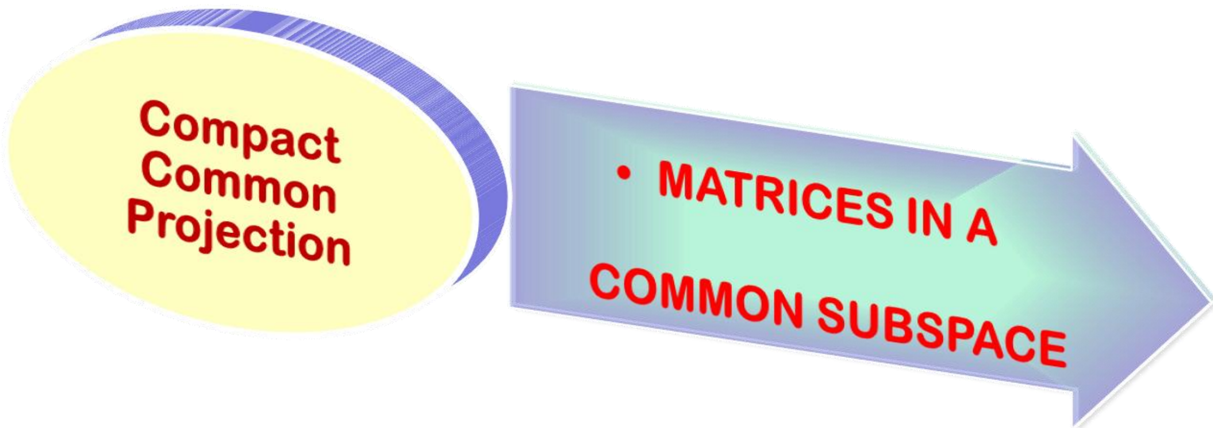
CONCLUSION



CONCLUSION

Compact Common Projection

- MATRICES IN A COMMON SUBSPACE

Congruence transformation

- SPECIFIC FORMAT

CONCLUSION

Compact Common Projection	• MATRICES IN A COMMON SUBSPACE
Congruence transformation	• SPECIFIC FORMAT
Positive Interpolation	• INTERPOLATION WITH RESPECT TO DESIGN SPACE

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P-ROM WITH GUARANTEED PASSIVITY USING MATRIX INTERPOLATION

THANK YOU