

# Frequency and Time Domain Variability Analysis of an On-Chip Inverted Embedded Microstrip Line Using a Macromodeling-based Stochastic Galerkin Method

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- **Motivation**
- **Interconnect example**
- **New stochastic modeling strategy**
- **Variability analysis of on-chip IEM line**
- **Conclusions**

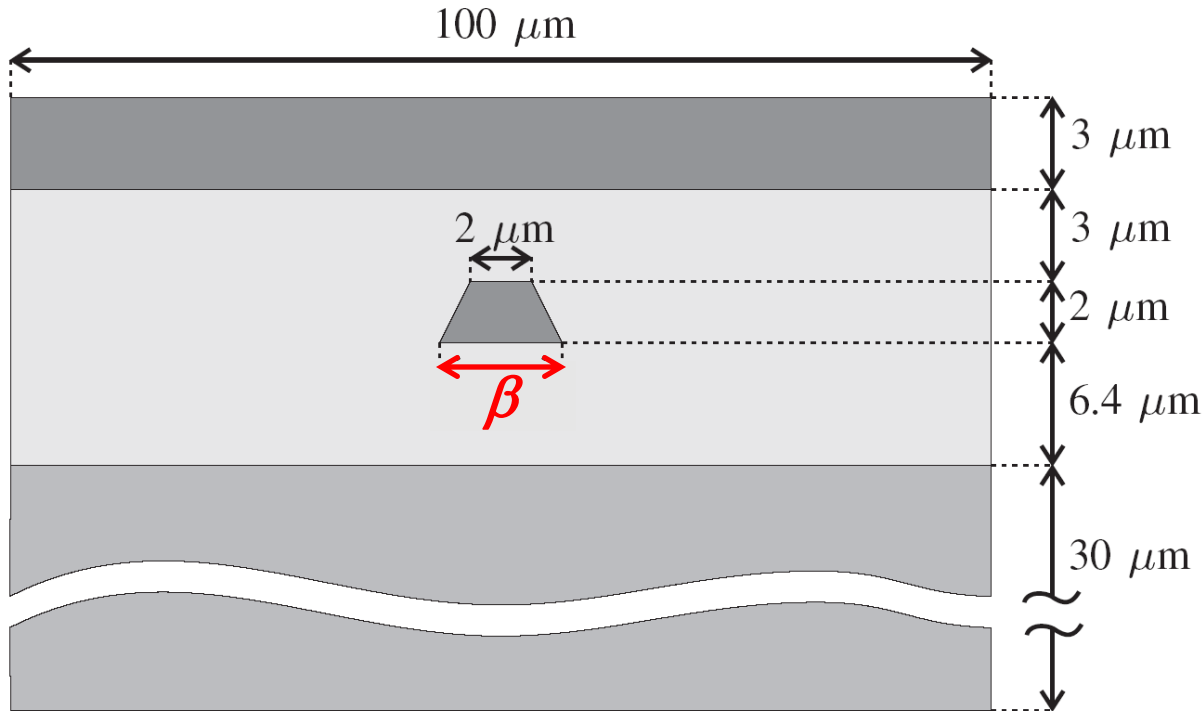
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## *Modeling for on-chip interconnect design*

- More and more **wave effects** appearing:
  - Interconnects become electrically long(er)
  - Skin-effect, slow-wave effect (semiconductors), ...
- Ever more stringent **design specifications**
  - Bandwidth, speed, crosstalk, noise margin, ...
- **Miniaturization**
  - Manufacturing process introduces randomness
  - Position and width, shape of cross-section, ...

***Designers need accurate modeling tools  
that allow an efficient variability analysis***

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Aluminum :  $\sigma = 3.77 \cdot 10^7 \text{ S/m}$

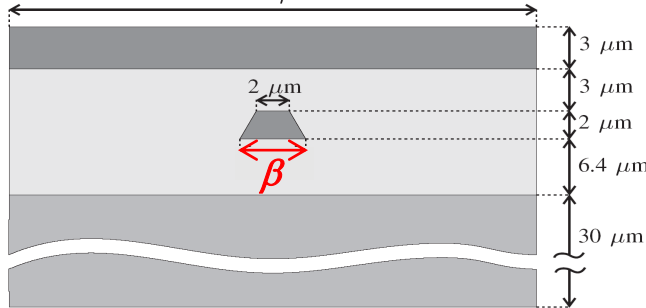
SiO<sub>2</sub> :  $\epsilon_r = 3.9$ ,  $\tan \delta = 0.001$

Silicon :  $\epsilon_r = 11.7$ ,  $\sigma = 10 \text{ S/m}$

→ **Inverted Embedded Microstrip (IEM) line**

→ **Due to manufacturing: random parameter  $\beta$**

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**2-D EM modeling**

*Tabulated p.u.l.  
parameter data*

**Macromodeling**

**Convergence?**

*Yes => Analytical expressions  
for p.u.l. parameters*

**Stochastic Galerkin Method**

**AFS**

*no*

## STEP 1: 2-D EM modeling

### ■ *Stochastic* Telegrapher's equations (*single line*):

$$\begin{aligned} \frac{d}{dz} V(z, s, \beta) &= -Z(s, \beta) I(z, s, \beta), \\ \frac{d}{dz} I(z, s, \beta) &= -Y(s, \beta) V(z, s, \beta) \end{aligned}$$

Diagram illustrating the stochastic Telegrapher's equations. Red question marks are placed under the derivative terms  $\frac{d}{dz}$  in both equations. Blue arrows point from the red question marks to the stochastic parameter  $\beta$  in the arguments of the functions  $V$  and  $I$ .

- $V$  and  $I$  : unknown voltages and currents along line
  - Function of position, frequency and of stochastic parameter  $\beta$
- $Z$  and  $Y$  : known p.u.l. transmission line parameters
  - UGent's Dirichlet-to-Neumann (DtN) solver
  - Tabulated data, but very accurate (**all loss mechanisms**)

## STEP 2: Parameterized macromodeling

### ■ Vector Fitting and Lagrange interpolation

$$Z^{\text{mm}}(s, \beta) = \sum_{v=1}^V w_v Z^{\text{umm}}(s, \beta_v) \prod_{\substack{k=1 \\ k \neq v}}^V (\beta - \beta_k),$$

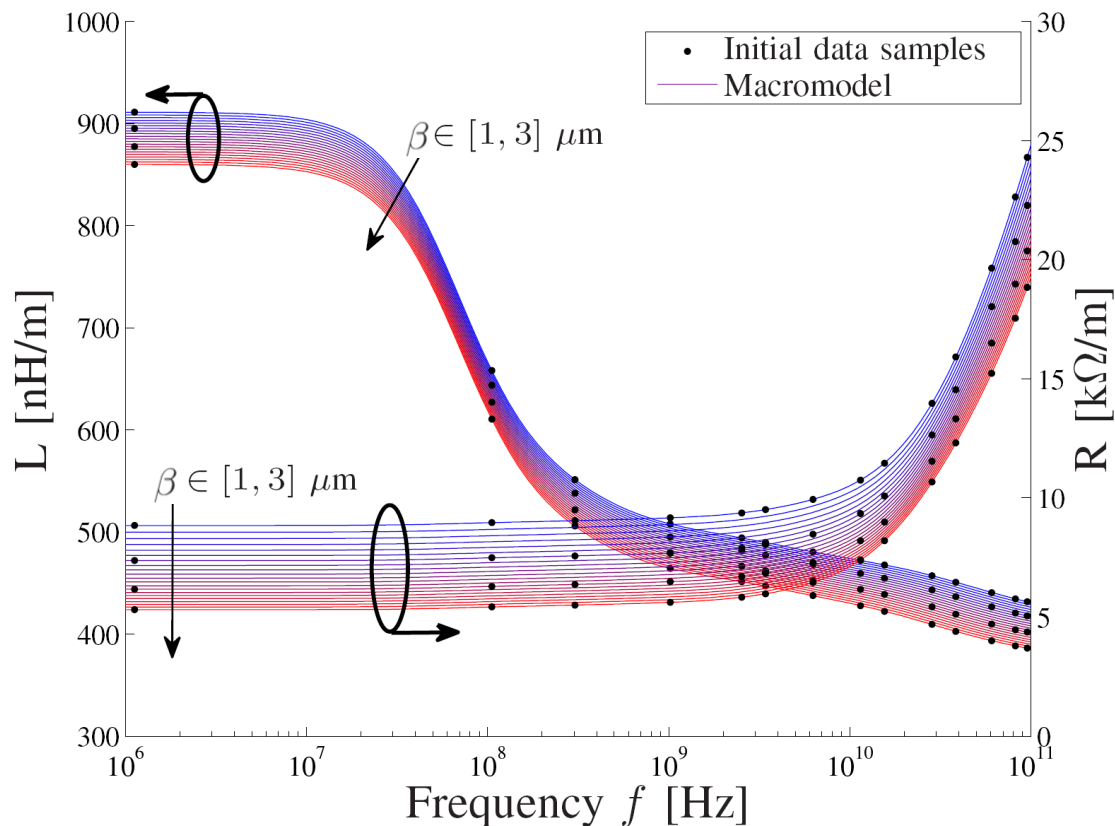
$$Y^{\text{mm}}(s, \beta) = \sum_{v=1}^V w_v Y^{\text{umm}}(s, \beta_v) \prod_{\substack{k=1 \\ k \neq v}}^V (\beta - \beta_k).$$

### ■ Rational w.r.t. frequency / polynomial w.r.t. $\beta$

- For all **on-chip interconnects** (no heuristic models)
- Polynomials  $\rightarrow$  convenient choice

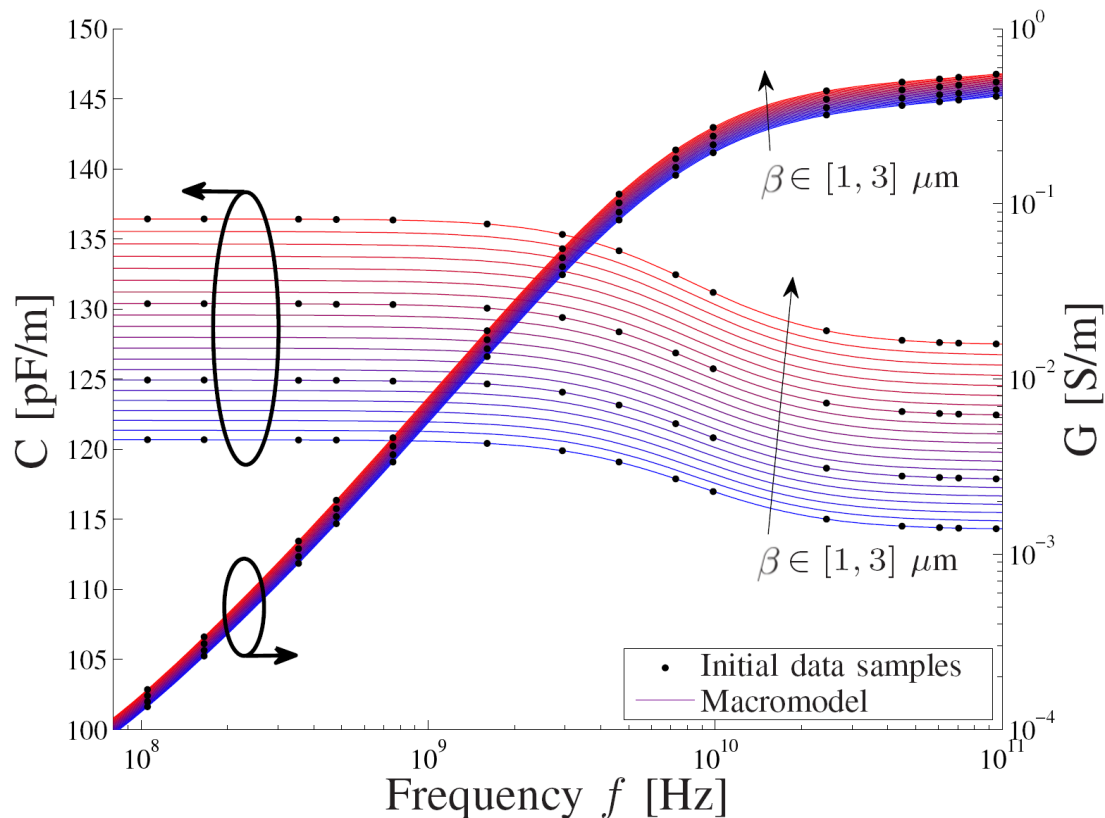
## Preliminary result (after step 1 and 2):

### ■ P.u.l. impedance ( $Z = R + sL$ )



## Preliminary result (after step 1 and 2) – cont.:

### ■ P.u.l. admittance ( $Y = G + sC$ )



## STEP 3: Stochastic Galerkin Method (SGM)

- Assume, e.g., that  $\beta$  is random variable
  - Normalize:  $\beta = \mu_\beta(1 + \sigma_\beta\xi)$
- SGM: Polynomial Chaos (PC) expansion + Galerkin weighting
  1. PC-expanded Telegrapher's equations :

$$\frac{d}{dz} \sum_{k=0}^K V_k(z, s) \phi_k(\xi) = - \sum_{k=0}^K \sum_{l=0}^K Z_k(s) I_l(z, s) \phi_k(\xi) \phi_l(\xi)$$

$$\frac{d}{dz} \sum_{k=0}^K I_k(z, s) \phi_k(\xi) = - \sum_{k=0}^K \sum_{l=0}^K Y_k(s) V_l(z, s) \phi_k(\xi) \phi_l(\xi)$$

The equations above contain red question marks and blue arrows indicating the stochastic Galerkin weighting process. In the first equation, red question marks are under  $V_k$  and  $I_l$ , and blue arrows point from  $\phi_k(\xi)$  and  $\phi_l(\xi)$  to the  $I_l$  term. In the second equation, red question marks are under  $I_k$  and  $V_l$ , and blue arrows point from  $\phi_k(\xi)$  and  $\phi_l(\xi)$  to the  $V_l$  term.

## STEP 3: Stochastic Galerkin method (SGM) – cont.

2. Galerkin projection, i.e. weighting with same set of polynomials  $\phi_m(\xi)$ ,  $m = 0, \dots, K$  :

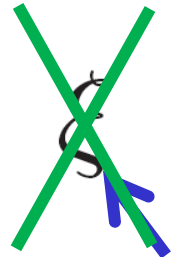
$$\forall m = 0, \dots, K,$$

$$\frac{d}{dz} V_m(z, s) = - \sum_{k=0}^K \sum_{l=0}^K \alpha_{klm} Z_k(s) I_l(z, s)$$

? ?

$$\frac{d}{dz} I_m(z, s) = - \sum_{k=0}^K \sum_{l=0}^K \alpha_{klm} Y_k(s) V_l(z, s)$$

? ?

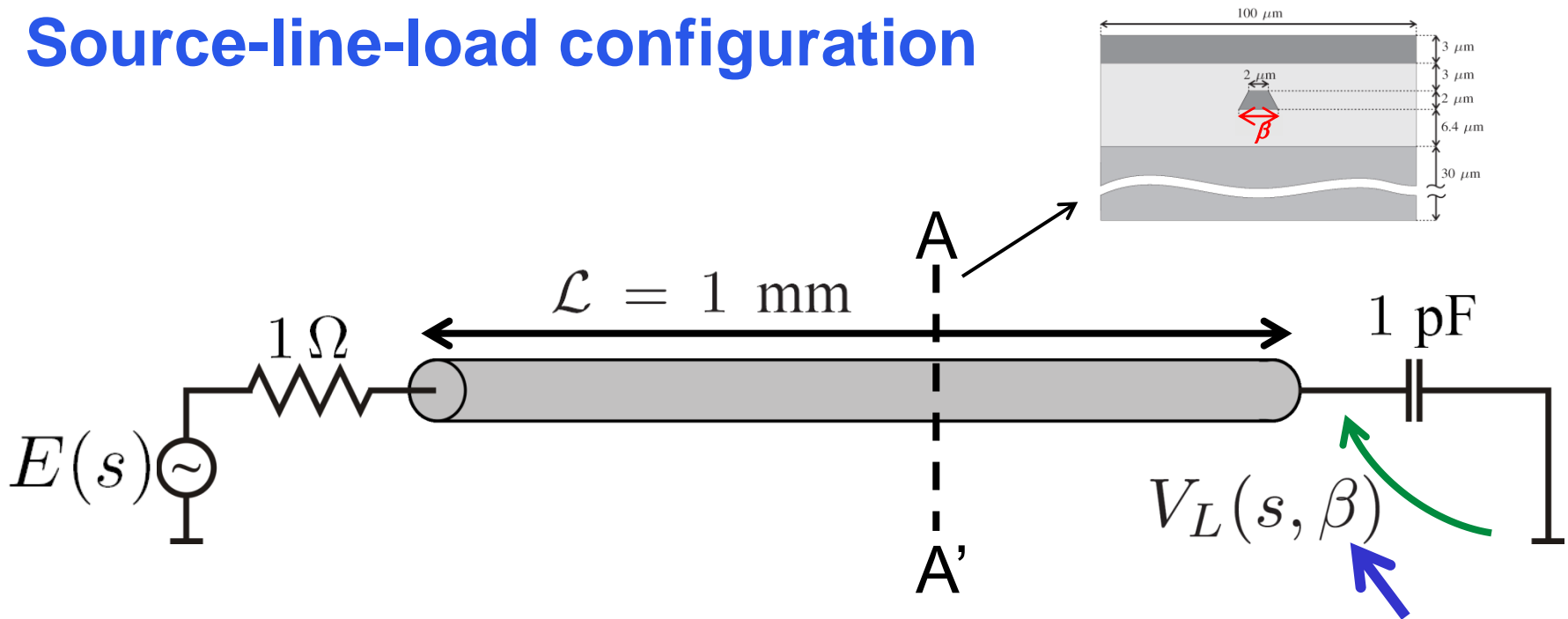


with:  $\alpha_{klm} = \langle \phi_k(\xi) \phi_l(\xi), \phi_m(\xi) \rangle / m!$

- **Thanks to SGM, this is a *deterministic* matrix ordinary differential equation (ODE)**

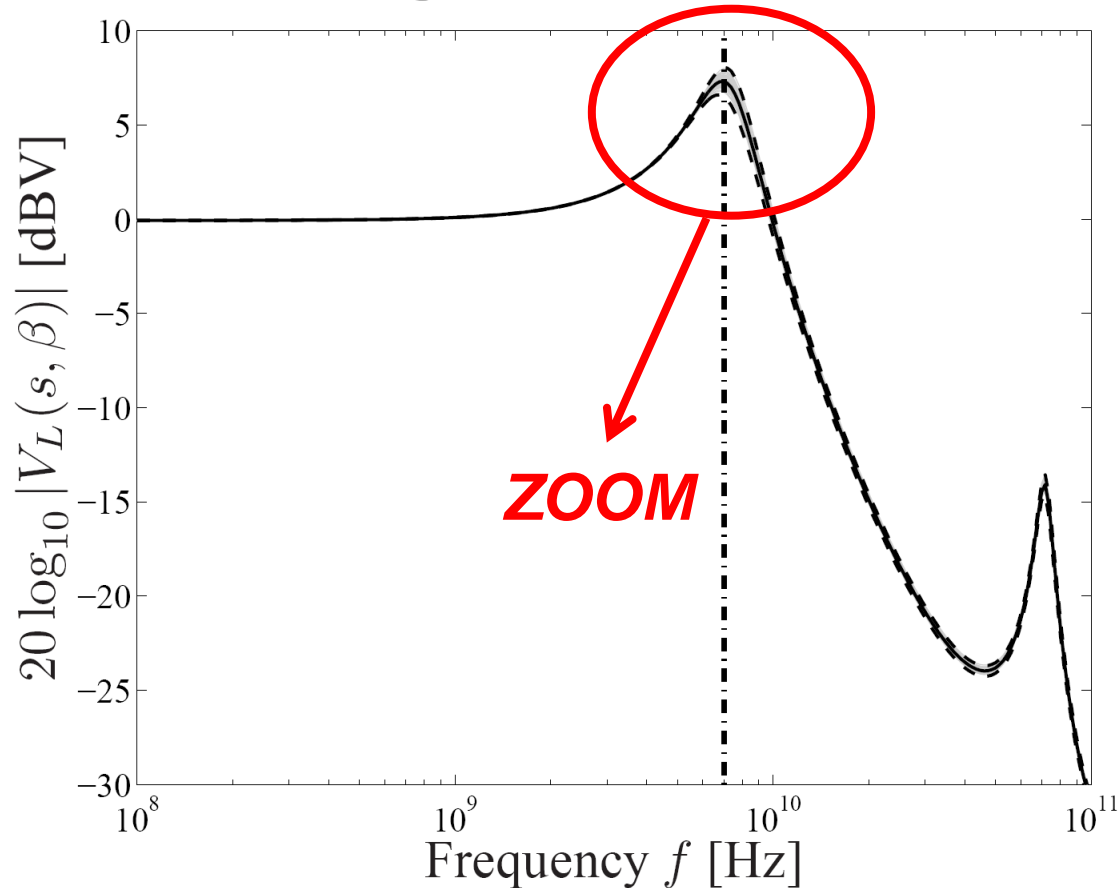
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## Source-line-load configuration



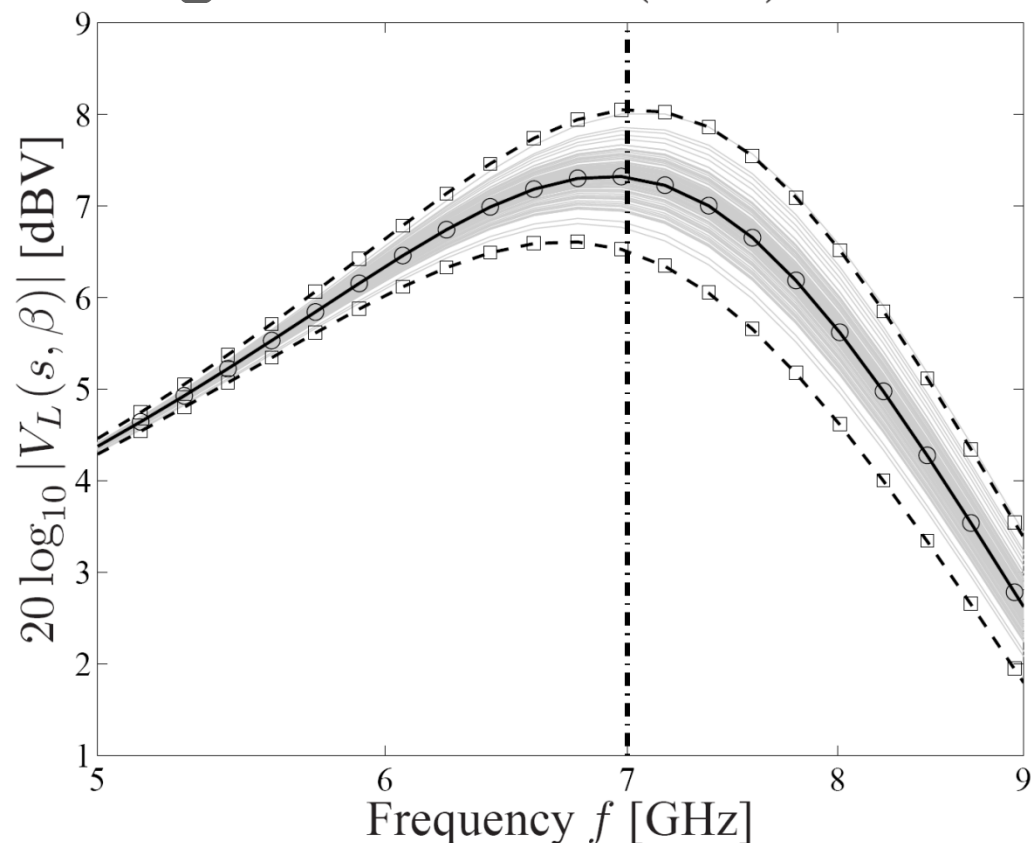
- $\beta$  is a Gaussian RV:  $\mu_\beta = 2\ \mu\text{m}$  and  $\sigma_\beta = 10\%$
- Post-processing: time domain (transient source)
- Compare with MC run (50000 samples of  $\beta$ )

## Voltage at load $V_L(s, \beta)$



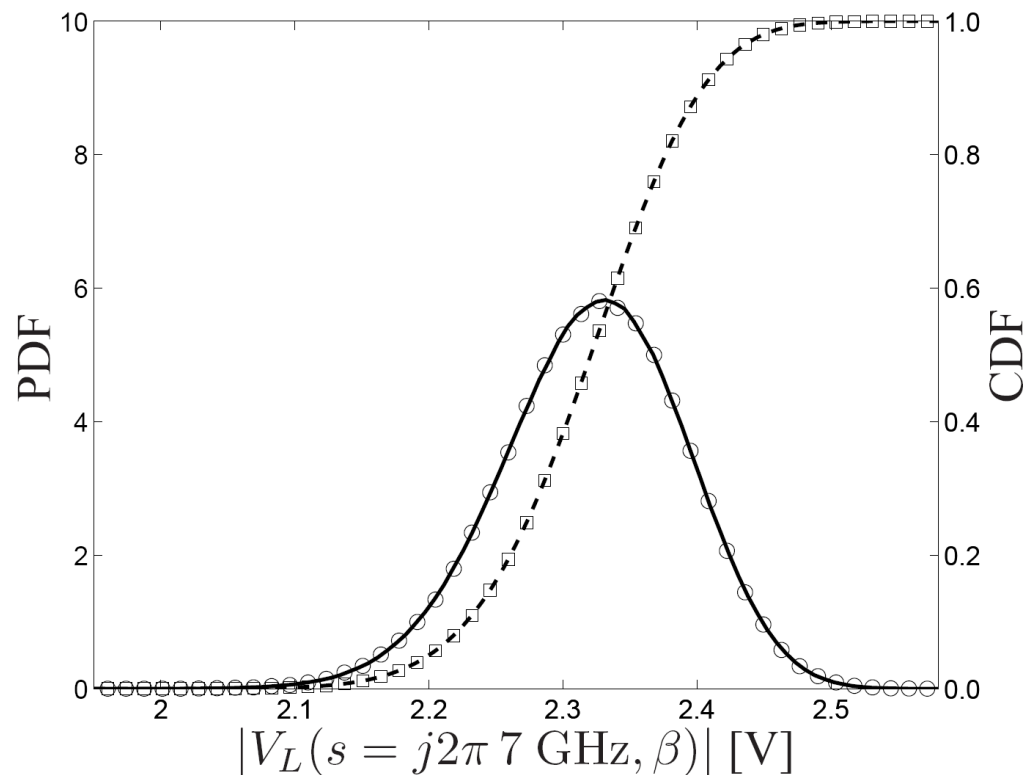
Full: mean  $\mu_{|V_L|}$  (SGM) / Dashed:  $\pm 3\sigma_{|V_L|}$  -variation (SGM)  
/ Gray: MC samples

## Voltage at load $V_L(s, \beta)$ : detail



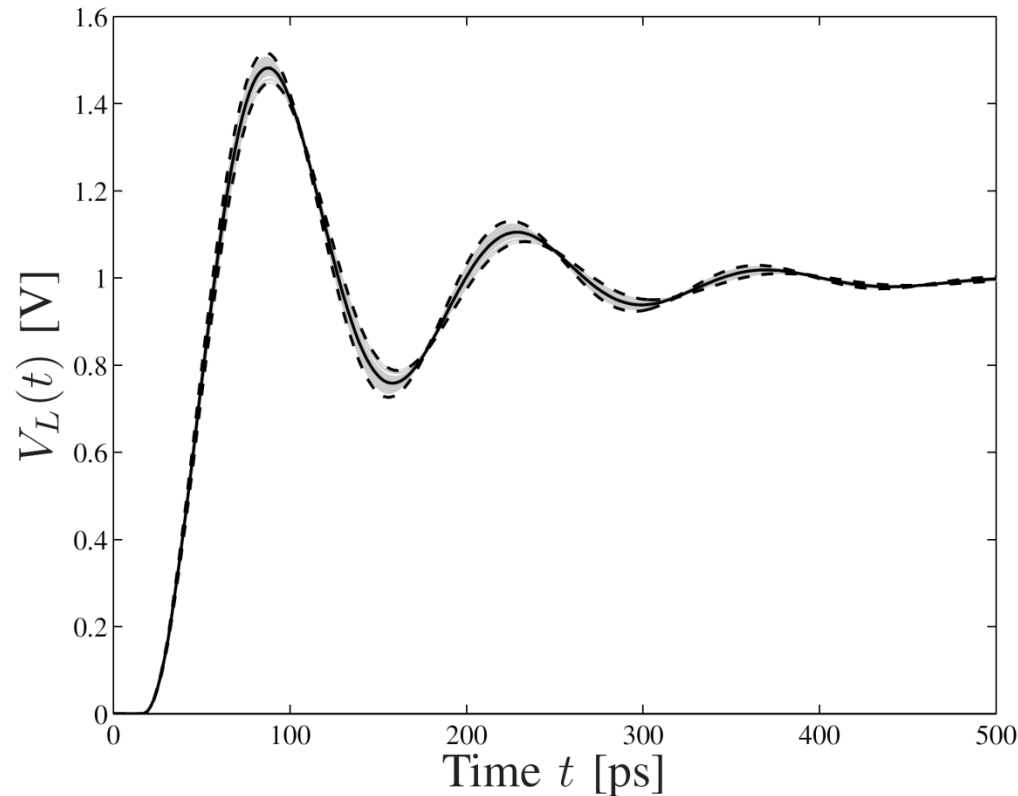
**Full: mean  $\mu_{|V_L|}$  (SGM) / Dashed:  $\pm 3\sigma_{|V_L|}$ -variation (SGM) /**  
**Gray: MC samples / Circles: mean  $\mu_{|V_L|}$  (MC) /**  
**Squares:  $\pm 3\sigma_{|V_L|}$ -variations (MC)**

## Probability density function (PDF) and cumulative distribution function (CDF) @ 7 GHz



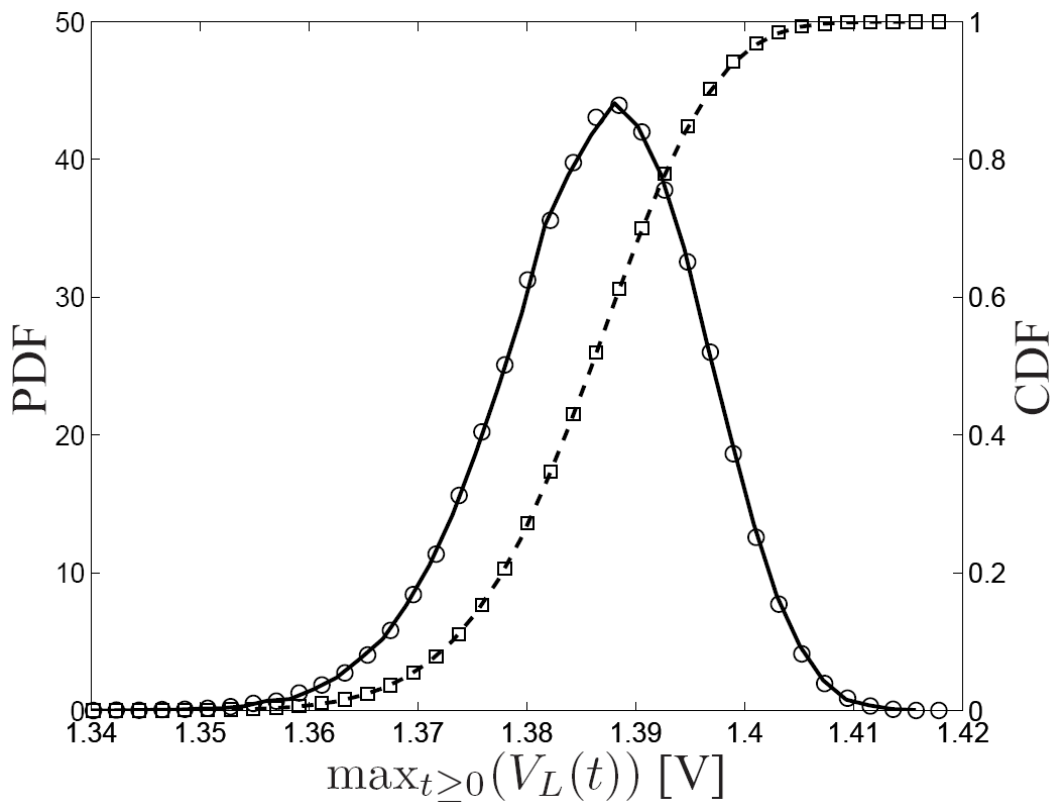
Full and dashed lines: SGM / Circles and squares: MC

## Voltage at load $V_L(t)$



**Time-domain voltage source: ramped step (0V  $\rightarrow$  1V), 30 ps rise time**

# Probability density function (PDF) and cumulative distribution function (CDF) of **overshoot**



Full and dashed lines: SGM / Circles and squares: MC

- **2-D EM + macromodeling (offline):**
  - Accuracy: 0.1%
  - CPU time: 140 s
- **Efficiency of novel stochastic modeling strategy compared to “tractable MC(\*)”:**

Technique	CPU time [s]			Speed-up factor
	setup	solve	total	
Novel approach	0.02	0.11	0.13	32
Monte Carlo			4.13	

***Note: MC without macromodeling takes about 6 months...***

(\*) D. Vande Ginste, D. De Zutter, D. Deschrijver, T. Dhaene, and F. Canavero, “Macromodeling based variability analysis of an inverted embedded microstrip line,” *IEEE 20<sup>th</sup> Conf. on EPEPS*, 23-26 Oct. 2011, pp. 153-156

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- **State-of-the-art on-chip interconnect design:**  
manufacturing process plays an important role
- **New (stochastic) modeling methods needed**
- **Presented stochastic modeling strategy**
  - 2-D EM modeling
  - Macromodeling
  - Stochastic Galerkin Method

**→ First method for *on-chip* interconnects!**

## ■ Tested on on-chip interconnects:

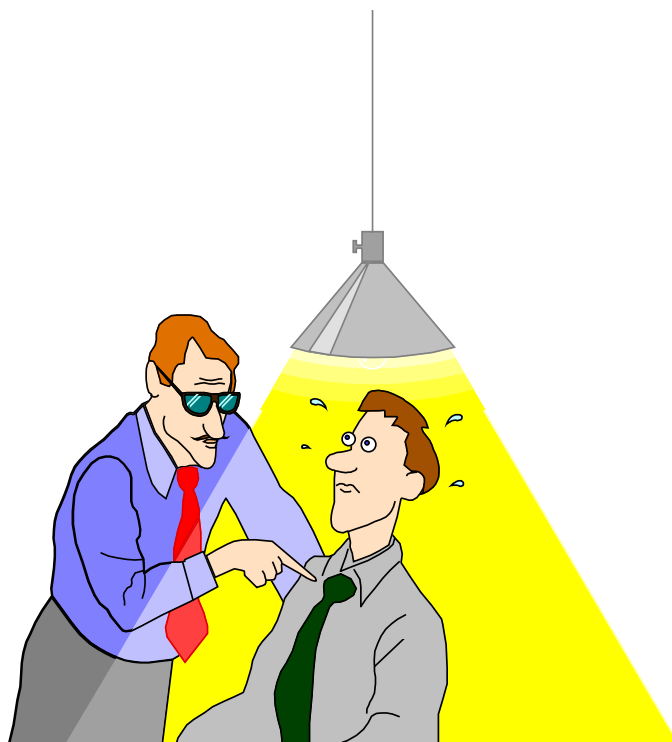
- Single IEM line
- Substrate losses, semiconductors, finite thickness and conductivity of the metallic line, ...
- Influence of manufacturing: trapezoidal cross-section

## ■ Comparison with MC:

- Excellent accuracy
- Much improved efficiency

## ■ Further reading:

D. Vande Ginste, D. De Zutter, D. Deschrijver, T. Dhaene, P. Manfredi, and F. Canavero, "Stochastic Modeling-Based Variability Analysis of On-Chip Interconnects," *IEEE Trans. on Components, Packaging, and Manufacturing Technology, IEEE Early Access*, 2012



## *Questions?*